RESULTATE

DER

RECHNUNGS-AUFGABEN IN DER SAMMLUNG

VON

AUFGABEN UND BEISPIELEN

AUS DER

TRIGONOMETRIE UND STEREOMETRIE

HERAUSGEGEBEN

VON

DR. FRIEDRICH REIDT,

OBERLEHRER AM GYMNASIUM UND DER HÖHEREN BÜRGERSCHULE IN HAMM.

I. THEIL: TRIGONOMETRIE.

ZWEITE AUFLAGE.

LEIPZIG,

DRUCK UND VERLAG VON B. G. TEUBNER.

1878.

Trigonometrische Aufgaben.

A. Goniometrie.

§. 1.

1. $1\frac{1}{2}$, $2\frac{1}{4}$, 3, $3\frac{3}{4}$ und $1\frac{1}{4}$, $2\frac{1}{2}$, $3\frac{3}{4}$, 5, $6\frac{1}{4}$. **2.** $7\frac{1}{2}$. **3.** 4:5, 3:5, 4:3 etc. **4.** $\sin\beta = \cos\alpha$, $\cos\beta = \sin\alpha$, $\operatorname{tang}\beta = \operatorname{cotg}\alpha$, etc.

5.

	sinus	cosinus	tangens	cotang.	secans	cosecans
a)	$\frac{8}{17}$ oder 0,47059	$\frac{15}{17}$ 0,88235	$\frac{8}{15}$ 0,53333	$\frac{15}{8}$ 1,87500	$\frac{17}{15}$ 1,13333	$\frac{17}{8}$ 2,12500
b)	$\frac{40}{41}$ 0,97561	$\frac{9}{41}$ 0,21951	$\frac{40}{9}$ 4,44444	$\frac{9}{40}$ 0,22500	$\frac{41}{9}$ 4,55556	$\frac{41}{40}$ 1,02500
c)	0,52830	0,84906	0,62222	1,60714	1,17778	1,89286
d)	$\frac{16}{65}$ 0,24615	$\frac{63}{65}$ 0,96923	$\frac{16}{63}$ 0,25397	$\frac{63}{16}$ 3,93750	$\frac{65}{63}$ 1,03175	$\frac{65}{16}$ 4,06250
e)	$\frac{80}{89}$ 0,89888	$\frac{39}{89}$ 0,43820	$\frac{80}{39}$ 2,05128	$\frac{39}{80}$ 0,48750	$\frac{89}{39}$ 2,28205	$\frac{89}{80}$ 1,11250
f)	0,86154	0,50769	1,69697	0,58929	1,96970	1,16071
g)	0,99792	0,06445	15,48387	0,06458	15,51613	1,00208
h)	$\frac{340}{389}$ 0,87404	$\frac{189}{389}$ 0,48586	$\frac{340}{189}$ 1,79894	$\frac{189}{340}$ 0,55588	$\frac{389}{189}$ 2,05820	$\frac{389}{340}$ 1,14412
i)	0,29340	0,95599	0,30691	3,25833	1,04604	3,40833
k)	0,15898	0,98728	0,16103	6,21000	1,01288	6,29000
l)	0,74844	0,66320	1,12853	0,88611	1,50784	1,33611
m)	$\frac{32}{257}$ 0,12451	$\frac{255}{257}$ 0,99222	$\frac{32}{255}$ 0,12549	$\frac{255}{32}$ 7,96875	$\frac{257}{255}$ 1,00784	$\frac{257}{32}$ 8,03125
n)	$\frac{204}{325}$ 0,62769	$\frac{253}{325}$ 0,77846	$\frac{204}{253}$ 0,80632	$\frac{253}{204}$ 1,24020	$\frac{325}{253}$ 1,28458	$\frac{325}{204}$ 1,59314
o)	$\frac{120}{409}$ 0,29340	$\frac{391}{409}$ 0,95599	$\frac{120}{391}$ 0,30691	$\frac{391}{120}$ 3,25833	$\frac{409}{391}$ 1,04604	$\frac{409}{120}$ 3,40833
p)	$\frac{12}{13}$ 0,92308	$\frac{5}{13}$ 0,38462	$\frac{12}{5}$ 2,40000	$\frac{5}{12}$ 0,41667	$\frac{13}{5}$ 2,60000	$\frac{13}{12}$ 1,08333
q)	$\frac{440}{521}$ 0,84453	$\frac{279}{521}$ 0,53551	$\frac{440}{279}$ 1,57706	$\frac{279}{440}$ 0,63409	$\frac{521}{279}$ 1,86738	$\frac{521}{440}$ 1,18409

Es muss $a^2 + b^2 = c^2$ sein, was hier auch überall der Fall ist.

6. a) $\frac{p}{r}, \frac{q}{r}, \frac{p}{q}, \frac{q}{p}, \frac{r}{q}, \frac{r}{p}$. b) $\frac{q}{r}, \frac{q}{p}, \frac{p}{r}, \frac{r}{p}, \frac{p}{q}, \frac{r}{q}$. c) $\frac{q}{s}, \frac{q}{p}, \frac{p}{s}, \frac{s}{p}, \frac{p}{q}, \frac{s}{q}$. d) $\frac{ps}{qr}, \frac{qs}{r^2}, \frac{pr}{q^2}, \frac{q^2}{pr}, \frac{r^2}{qs}, \frac{rq}{ps}$. e) $\frac{m(m+n)}{p(p+q)}, \frac{m(n+p)}{q(p+q)}, \frac{q(m+n)}{p(n+p)}, \frac{p(n+p)}{q(m+n)}$, u. s. w. f) $\frac{ms}{qr}, \frac{mp}{nq}, \frac{ns}{pr}, \frac{pr}{ns}$. g) $\frac{2mn}{m^2+n^2}, \frac{m^2-n^2}{m^2+n^2}, \frac{2mn}{m^2-n^2}, \frac{m^2-n^2}{2mn}$. h) $\frac{2mn}{m^2+n^2}, \frac{m^2-n^2}{m^2+n^2}, \frac{2mn}{m^2-n^2}$. i) $\frac{m^2-n^2}{\sqrt{2(m^4+n^4)}}, \frac{m^2+n^2}{\sqrt{2(m^4+n^4)}}, \frac{m^2-n^2}{m^2+n^2}, \frac{m^2+n^2}{m^2-n^2}$. k) $\frac{m-n}{m+n}, \frac{2\sqrt{mn}}{m+n}, \frac{m-n}{2\sqrt{mn}}, \frac{2\sqrt{mn}}{m-n}$. l) $\frac{p-q}{p+q}, \frac{2\sqrt{pq}}{p+q}, \frac{p-q}{2\sqrt{pq}}, \frac{2\sqrt{pq}}{p-q}$. m) $\left(\frac{m-n}{m+n}\right)^2, \frac{2\sqrt{mn}}{(m+n)^2}, \frac{(m-n)^2}{2\sqrt{mn}}, \frac{2\sqrt{mn}}{(m-n)^2}$. Frage, wie vorher.

7. a) $c = 6{,}29$; $\frac{100}{629}, \frac{621}{629}, \frac{100}{621}, \frac{621}{100}$. b) $c = 4{,}93$; $\frac{468}{493}$; $\frac{155}{493}, \frac{468}{155}, \frac{155}{468}$. c) $c = 46{,}1$; $a : b : c = 380 : 261 : 461$. d) $c = 8{,}81$; $a : b : c = 800 : 369 : 881$. e) $b = 64{,}6$; $36 : 323 : 325$. f) $b = 3{,}01$; $900 : 301 : 949$. g) $b = 0{,}49$; $1200 : 49 : 1201$. h) $b = 31\frac{7}{8}$; $32 : 255 : 257$. i) $a = 39{,}9$; $399 : 40 : 401$. k) $a = 4{,}29$; $429 : 700 : 821$. l) $a = 4{,}81$; $481 : 600 : 769$.

8. a) $c = m + n$; $a : b : c = \sqrt{m^2+n^2} : \sqrt{2mn} : (m+n)$.

b) $c = p - q$; $\sqrt{p^2 - 2pq} : q : (p - q)$.

c) $b = \sqrt{q(p+q)}$; $\sqrt{\frac{p}{p+q}}, \sqrt{\frac{q}{p+q}}, \sqrt{\frac{p}{q}}$.

d) $c = \sqrt{2(p^2+q^2)}$; $(p-q) : (p+q) : \sqrt{2(p^2+q^2)}$.

e) $b = (m+n)\sqrt{m^2-n^2}$; $\frac{n}{m}, \frac{\sqrt{m^2-n^2}}{m}, \frac{n}{\sqrt{m^2-n^2}}$.

f) $a = \frac{p}{qr}\sqrt{q^2-r^2}$; $\frac{\sqrt{q^2-r^2}}{q}, \frac{r}{q}, \frac{\sqrt{q^2-r^2}}{r}$.

g) $a = \frac{p}{qr}\sqrt{r^4-q^4}$; $\frac{\sqrt{r^4-q^4}}{r^2}, \frac{q^2}{r^2}, \frac{\sqrt{r^4-q^4}}{q^2}$.

h) $a = n(m-n)$; $n : \sqrt{m^2-n^2} : m$.

i) $b = pq\sqrt{2}$.

9. $\sin 45^0 = \cos 45^0 = \frac{1}{2}\sqrt{2} = 0{,}70711$; $\tang 45^0 = \cotg 45^0 = 1$.

$\sin 30^0 = \cos 60^0 = 0{,}5$; $\cos 30^0 = \sin 60^0 = \frac{1}{2}\sqrt{3} = 0{,}86603$;

$\tan 30^0 = \cot 60^0 = \frac{1}{3}\sqrt{3} = 0{,}57735$; $\cot 30^0 = \tan 60^0 = \sqrt{3} = 1{,}73205$.

$\sin 18^0 = \cos 72^0 = \frac{1}{4}(\sqrt{5} - 1) = 0{,}30902$; $\cos 18^0 = \sin 72^0 = \frac{1}{4}\sqrt{10 + 2\sqrt{5}} = 0{,}95106$; $\tan 18^0 = \cot 72^0 = \sqrt{\frac{5 - 2\sqrt{5}}{5}} = 0{,}32492$; $\cot 18^0 = \tan 72^0 = \sqrt{5 + 2\sqrt{5}} = 3{,}07768$.

10. a) $\sin\alpha = \frac{2}{5}\sqrt{5} = 0{,}89443$; $\cos\alpha = \frac{1}{5}\sqrt{5} = 0{,}44721$; $\tan\alpha = 2$; $\cot\alpha = \frac{1}{2}$.

b) $\sin\alpha = \frac{2}{3} = 0{,}66667$; $\cos\alpha = \frac{1}{3}\sqrt{5} = 0{,}74536$; $\tan\alpha = \frac{2}{5}\sqrt{5} = 0{,}89443$; $\cot\alpha = \frac{1}{2}\sqrt{5} = 1{,}11803$.

c) $\sin\alpha = \frac{1}{8}(5 + \sqrt{7}) = 0{,}95572$; $\cos\alpha = \frac{1}{8}(5 - \sqrt{7}) = 0{,}29428$; $\tan\alpha = \frac{1}{9}(16 + 5\sqrt{7}) = 3{,}24764$; $\cot\alpha = \frac{1}{9}(16 - 5\sqrt{7}) = 0{,}30792$.

d) $\sin\alpha = \frac{\sqrt{2m^2 - 1} + 1}{2m}$, $\cos\alpha = \frac{\sqrt{2m^2 - 1} - 1}{2m}$, $\tan\alpha = \frac{m^2 + \sqrt{2m^2 - 1}}{m^2 - 1}$, $\cot\alpha = \frac{m^2 - \sqrt{2m^2 - 1}}{m^2 - 1}$.

11. a) $\sin\alpha = \sqrt{a^2 - h^2} : a$, $\cos\alpha = h : a$, $\tan\alpha = \sqrt{a^2 - h^2} : h$, $\cot\alpha = h : \sqrt{a^2 - h^2}$.

b) $\sin\alpha = \sqrt{p : (p + q)}$, $\cos\alpha = \sqrt{q : (p + q)}$, $\tan\alpha = \sqrt{p : q}$, $\cot\alpha = \sqrt{q : p}$.

c) $\sin\alpha = \frac{\sqrt{4a^2 + q^2} - q}{2a}$, $\cos\alpha = \frac{\sqrt{\frac{1}{2}q(\sqrt{4a^2 + q^2} - q)}}{a}$, u. s. w.

d) $\sin\alpha = \frac{\sqrt{c^2 + 4F} + \sqrt{c^2 - 4F}}{2c}$, $\cos\alpha = \frac{\sqrt{c^2 + 4F} - \sqrt{c^2 - 4F}}{2c}$, $\tan\alpha = \frac{c^2 + \sqrt{c^4 - 16F^2}}{4F}$, $\cot\alpha = \frac{c^2 - \sqrt{c^4 - 16F^2}}{4F}$.

12. a) 12,3. b) 1,54. c) 9. d) 0,03804. e) 128,38384. f) 3,22581.

13. $a = 6$, $b = 4$. **14.** a) $a = 1{,}05$, $b = 1$, $F = 0{,}525$. b) $c = 25$, $b = 7$, $F = 84$. c) $c = 125$, $a = 44$, $F = 2574$.

18. $a = 0{,}087$; $b = 0{,}996$; $a = 0{,}176$; $b = 3{,}732$.

§. 2.

5. 45^0. **6.** $\frac{180^0}{n}$. **7.** Vergl. §. 1, 9. **12.** $r \cdot a$. **13.** $a = mc$; $b = nc$. **14.** a) $S = r.s$; b) $\frac{r^2 \pi \alpha}{360^0} - \frac{1}{4} r^2 s \sqrt{4 - s^2}$, wenn s die zugehörige Sehne der Tafel bedeutet. **15.** $\sin \frac{1}{2} \alpha = \frac{1}{2}$ Sehne α. **16.** $\sin \frac{1}{2} \alpha = \sqrt{\frac{1 - \cos \alpha}{2}}$, $\operatorname{tang} \alpha = \frac{\sin \alpha}{\cos \alpha}$.

§. 3.

1. Sinus, Tangente und Secante.

2. Sinus und Cosecante, Tangente und Cotangente, Cosinus und Secante.

3. Cosinus und Secante.

4. 4. **5.** 2, im ersten und vierten. **6.** Für $\sin \alpha$ zwei, sonst einer.

7. Sinus und Cosecante zweideutig, die übrigen Functionen eindeutig.

8. Cosinus, Tangente, Cotangente und Secante für stumpfe Winkel.

9. a) 0, b) 0, c) $(a - b)^2$, d) 0, e) $a^2 - b^2 + 4ab$, f) $(m - n)^2$, g) ∞, h) 0, i) $2 \frac{a^2 + b^2}{a^2 - b^2}$, k) $- xy$, l) $(a + b)^3$, m) 0, n) ab, o) b.

10. 1, 0, 0, ∞, 1, ∞.

11.

1. $\sin 8^0 = \cos 82^0$
2. $- \cos 80^0 = - \sin 10^0$
3. $- \operatorname{tg} 55^0 = - \operatorname{cotg} 35^0$
4. $- \operatorname{cotg} 85^0 = - \operatorname{tg} 5^0$
5. $- \cos 1^0 = - \sin 89^0$
6. $\sin 35^0 = \cos 55^0$
7. $- \operatorname{tg} 81^0 = - \operatorname{cotg} 9^0$
8. $- \sec 80^0 = - \operatorname{cosec} 10^0$
9. $\operatorname{cosec} 23^0 = \sec 67^0$
10. $- \operatorname{cotg} 89^0 = - \operatorname{tg} 1^0$
11. $- \operatorname{tang} 65^0\ 2' = - \operatorname{cotg} 24^0\ 58'$
12. $- \cos 47.49 = - \sin 42.11$
13. $\operatorname{cosec} 80.31 = \sec 9.29$
14. $\sin 44.48 = \cos 45.12$
15. $- \operatorname{cotg} 40.43 = - \operatorname{tg} 49.17$
16. $\sin 35^0\ 50'\ 27'' = \cos 54^0\ 9'\ 33''$
17. $- \operatorname{tg} 27.11.53 = - \operatorname{cotg} 62.48.7$

18. $- \cot 10.\ 38.\ 12 = - \operatorname{tg} 79.\ 21.\ 48$
19. $- \cos 75.\ 8.\ 49 = - \sin 14.\ 51.\ 11$
20. $\sin 57.\ 37.\ 43 = \cos 32.\ 22.\ 17$
21. $- \cos 83^0\ 37'\ 48'',6 = - \sin 6^0\ 22'\ 11'',4$
22. $- \operatorname{tang} 83.\ 34.\ 59,3 = - \operatorname{cotg} 6.\ 25.\ 0,7$
23. $\sin 46.\ 10.\ 38,2 = \cos 43.\ 49.\ 21,8$
24. $- \operatorname{cotg} 58.\ 58.\ 12,4 = - \operatorname{tg} 31.\ 1.\ 47,6$
25. $- \cos 52.\ 0.\ 0,1 = - \sin 37.\ 59.\ 59,9$

26. $-\cos 14^0 = -\sin 76^0$ 29. $-\sin 24^0 12' = -\cos 65^0 48'$
27. $-\operatorname{tg} 60^0 = -\operatorname{cotg} 30^0$ 30. $\operatorname{cotg} 84.39 = \operatorname{tg} 5.\ 21$
28. $-\operatorname{cosec} 89^0 = -\sec 1^0$ 31. $-\sec 65.19 = -\operatorname{cosec} 24.\ 41$

32. $\sec 15^0\ 41'28'' = \operatorname{cosec} 74^0\ 18'32$
33. $\operatorname{tg} 32.\ 48.\ 44 = \operatorname{cotg} 57.\ 11.\ 16$
34. $\cos 85.\ 50.\ 8 = \sin 4.\ 9.\ 52$
35. $- \sin 76^0\ 10'\ 28'' = - \cos 13^0\ 49'\ 32''$
36. $- \operatorname{cosec} 1.\ 1.\ 1 = - \sec 88.\ 58.\ 59$
37. $\operatorname{tang} 89.\ 15.\ 15 = \operatorname{cotg} 0.\ 44.\ 45$
38. $- \operatorname{cotg} 86^0\ 8'\ 40'',6 = - \operatorname{tang} 3^0\ 51'\ 19'',4$
39. $- \cos 15.\ 33.\ 7,7 = - \sin 74.\ 26.\ 52,3$
40. $- \sin 9.\ 49.\ 38,1 = - \cos 80.\ 10.\ 21,9.$

12. a) $(a-b)\sin\alpha$; b) $m\sin\alpha\cos\alpha$; c) $(a-b)\operatorname{cotg}\alpha - (a+b)\operatorname{tang}\alpha$; d) $a^2+b^2+2ab\cos\gamma$; e) $\frac{a\sin(\alpha+\beta)}{\sin\alpha}$; f) $-\cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$; g) $\cos\alpha\cdot\sin\beta - \sin\alpha\,.\cos\beta$; h) $\operatorname{tang}\alpha - 2\operatorname{tang}\beta$; i) $\frac{a\sin\alpha - b\cos\alpha}{(b-a)\operatorname{cotg}\alpha}$; k) $\frac{m^3\operatorname{cotg}\alpha}{p^2 q\cos\alpha} - \frac{pq^2\cos\alpha}{m^3\operatorname{tang}\alpha}$; l) n; m) $2b^2$; n) 0; o) $\frac{\operatorname{cotg}\alpha}{\operatorname{tang}\alpha}$; p) -2; q) $-\sin\alpha$; r) $\cos\alpha - \frac{\operatorname{tang}\alpha}{\cos\alpha}$.

14. a) 168^0, b) 96^0, c) 142^0, d) 334^0, e) 225^0, f) 252^0, g) 240^0, h) 271^0, i) 349^0, k) $329^0\ 51'$, l) $180^0 - \alpha$.

15. Im a) dritten, b) zweiten, c) dritten.

16. a) $22^0\ 30'$, b) m; n, c) m.

17. Positiv für 0^0 bis 135^0 und 315^0 bis 360^0.

18. Ebenso für 45^0 bis 225^0.

19. Ebenso für $45^0 - 135^0$ und $225^0 - 315^0$.

20. Ebenso für $0^0 - 90^0$, $180^0 - 270^0$, bezw. $0^0 - 90^0$, $180^0 - 270^0$, bezw. $45^0 - 90^0$, $135^0 - 180^0$, $225^0 - 270^0$, $315^0 - 360^0$, bezw. $0^0 - 90^0$, $270^0 - 360^0$.

§. 4.

1. Die gesuchten Functionen sind in der Reihenfolge sin, cos, tang, cotg bezüglich gleich: **a)** 0,6; 1,333 ..., 0,75. **b)** $\frac{12}{13} = 0{,}92308$, $\frac{5}{13} = 0{,}38462$, $\frac{5}{12} = 0{,}41667$. **c)** 0,96; $\frac{24}{7} = 3{,}42857$; $\frac{7}{24} = 0{,}29167$. **d)** $\frac{20}{29} = 0{,}68966$, $\frac{21}{29} = 0{,}72414$, $\frac{20}{21} = 0{,}95238$. **e)** $\frac{99}{101}$, $\frac{20}{99}$, $\frac{99}{20} = 4\frac{19}{20}$. **f)** $\frac{221}{229}$, $\frac{60}{221}$, $3\frac{41}{60}$. **g)** $\frac{40}{41}$, $4\frac{4}{9}$, $\frac{9}{40}$. **h)** $\frac{399}{401}$, $9\frac{39}{40}$, $\frac{40}{399}$. **i)** $\frac{144}{145}$, $\frac{17}{145}$, $\frac{17}{144}$. **k)** $\frac{33}{545}$, $\frac{544}{545}$, $\frac{33}{544}$.

2. a) $\pm 0{,}936$; $\mp 0{,}37607$; $\mp 2{,}65909$. **b)** $\pm 0{,}97161$; $\mp 0{,}23659$; $-0{,}24351$. **c)** $\frac{2}{5}\sqrt{6} = \pm 0{,}97980$; $\mp 4{,}89898$; $\mp 0{,}20412$. **d)** $\frac{1}{4}\sqrt{7} = \pm 0{,}66144$; $\pm 1{,}13389$; $\pm 0{,}88192$. **e)** $\frac{1}{3}\sqrt{5} = \pm 0{,}74536$; $\pm 1{,}11803$; $\pm 0{,}89443$. **f)** $\frac{2}{5}\sqrt{5} = \pm 0{,}89443$; $\pm 0{,}44721$; 2.

3. a) $1 : \sqrt{1 + m^2}$, m, $1 : m$. **b)** $b : a$, $b : \sqrt{a^2 - b^2}$, $\sqrt{a^2 - b^2} : b$. **c)** $q : p$; $\sqrt{p^2 - q^2} : p$, $q : \sqrt{p^2 - q^2}$, $\sqrt{p^2 - q^2} : q$. **d)** $\frac{m-n}{m+n}$, $\frac{2\sqrt{mn}}{m+n}$, etc. **e)** $\frac{1}{\sqrt{1+a^2}}$, $\frac{a}{\sqrt{1+a^2}}$, a. **f)** $\frac{n}{m}$, $\frac{\sqrt{m^2-n^2}}{m}$, $\frac{n}{\sqrt{m^2-n^2}}$.

4. a) $\sin\alpha - \sin\alpha^3 + \frac{\sin\alpha}{1-\sin\alpha^2} - \frac{1}{\sin\alpha^2}$. **b)** $1 + \sin\alpha - \frac{\sin\alpha}{1-\sin\alpha^2}$. **c)** $\frac{\sin\alpha^2}{1-\sin\alpha^2} + 1 - \frac{1}{\sin\alpha^2} + 2\sin\alpha \,.\, \sqrt{1-\sin\alpha^2} = \frac{1}{1-\sin\alpha^2} - \frac{1}{\sin\alpha^2} + 2\sin\alpha \,.\, \sqrt{1-\sin\alpha^2}$. **d)** $\frac{1}{1-\sin\alpha^2} - \frac{\sin\alpha^2}{1-\sin\alpha^2} + 1 - \sin\alpha^2 - \frac{1-\sin\alpha^2}{\sin\alpha^2} = 2 - \sin\alpha^2 - \frac{1-\sin\alpha^2}{\sin\alpha^2}$.

5. a) $\frac{1-\cos\alpha^2}{\cos\alpha} + \frac{1-\cos\alpha^2}{\cos\alpha^2} = \frac{1+\cos\alpha-\cos\alpha^2-\cos\alpha^3}{\cos\alpha^2}$. **b)** $\frac{1}{\cos\alpha}$. **c)** $\frac{\cos\alpha^2}{1-\cos\alpha^2} + \frac{1-\cos\alpha^2}{\cos\alpha^2} - 1$. **d)** $\sqrt{1-\cos\alpha^2} \,.\, \cos\alpha$.

6. a) $\frac{1}{\operatorname{tang}\alpha} + \sqrt{1+\operatorname{tang}\alpha^2} - \frac{\sqrt{1+\operatorname{tang}\alpha^2}}{\operatorname{tang}\alpha}$. **b)** $\frac{\operatorname{tang}\alpha^2}{1+\operatorname{tang}\alpha^2} + \frac{1}{\operatorname{tang}\alpha} - \operatorname{tang}\alpha$. **c)** $\frac{\operatorname{tang}\alpha}{1+\operatorname{tang}\alpha^2} + \frac{1+\operatorname{tang}\alpha}{\sqrt{1+\operatorname{tang}\alpha^2}}$. **d)** $\frac{1-\operatorname{tang}\alpha+\operatorname{tang}\alpha^2}{1+\operatorname{tang}\alpha+\operatorname{tang}\alpha^2} - \frac{1}{\sqrt{1+\operatorname{tang}\alpha^2}}$.

7. a) 1. **b)** $2\operatorname{cotg}\alpha$. **c)** $\frac{1}{\operatorname{cotg}\alpha} + \operatorname{cotg}\alpha^2$.

8. a) $\sec\beta^2 - \frac{1}{\sec\beta^2 - 1}$. b) $1 + \sec\beta$. c) $\frac{1}{\sec\beta^2}$.

9. a) $\frac{1 + \sqrt{\operatorname{cosec}\gamma^2 - 1}}{\operatorname{cosec}\gamma}$. b) $\frac{1}{\sqrt{\operatorname{cosec}\gamma^2 - 1}} - \sqrt{\operatorname{cosec}\gamma^2 - 1}$.
c) $\frac{\operatorname{cosec}\gamma + 1}{\sqrt{\operatorname{cosec}\gamma^2 - 1}} + \frac{\sqrt{\operatorname{cosec}\gamma^2 - 1}}{\operatorname{cosec}\gamma}$.

10. a) $\sin x = \frac{1}{3} = 0{,}333\ldots$; $\cos x = \frac{2}{3}\sqrt{2} - 0{,}94281$; $\operatorname{tang} x = \frac{1}{4}\sqrt{2} = 0{,}35355$; $\operatorname{cotg} x = 2\sqrt{2} = 2{,}82843$.

b) $\cos x = \frac{2}{3}$ oder -1. Vergl. 2, e.

c) $\sin x = \pm\frac{1}{2}\sqrt{2} = \cos x$; $\operatorname{tang} x = \pm 1 = \operatorname{cotg} x$.

d) $\operatorname{tang} x = \frac{1}{2}\sqrt{3} = \pm 0{,}86603$; $\operatorname{cotg} x = \frac{2}{3}\sqrt{3} = \pm 1{,}15470$; $\cos x = \frac{2}{7}\sqrt{7} = \pm 0{,}75593$; $\sin x = \frac{1}{7}\sqrt{21} = \pm 0{,}65465$.

e) $\sin x = \frac{527}{625}$ oder -1; $\cos x_1 = \frac{336}{625}$; $\operatorname{tang} x_1 = \frac{527}{336}$.

f) $\operatorname{tang} x = \frac{9}{10}$ oder $\frac{1}{2}$; $\sin x = \frac{9}{\sqrt{181}} = 0{,}66896$ oder $\frac{1}{5}\sqrt{5} = 0{,}44721$; $\cos x = \frac{10}{\sqrt{181}} = 0{,}74329$ od. $\frac{2}{5}\sqrt{5} = 0{,}89443$; $\operatorname{cotg} x = \frac{10}{9} = 1{,}11111$ oder 2.

g) $\cos x = \frac{1}{3}$; $\sin x = \frac{2}{3}\sqrt{2} = 0{,}94281$; $\operatorname{tang} x = 2\sqrt{2} = 2{,}82843$; $\operatorname{cotg} x = \frac{1}{4}\sqrt{2} = 0{,}35355$.

h) $\sec x = 2$, $\cos x = \frac{1}{2}$, $\sin x = \frac{1}{2}\sqrt{3} = 0{,}86603$, $\operatorname{tang} x = \sqrt{3} = 1{,}73205$, $\operatorname{cotg} x = \frac{1}{3}\sqrt{3} = 0{,}57735$.

11. a) $\sin\alpha = \pm\frac{40}{41}$, $\cos\alpha = \pm\frac{9}{41}$.
b) $\sin\alpha = \frac{1}{5}\sqrt{5} = \pm 0{,}44721$.

59. a) $\cos 105^0 = -\sin 15^0$; $\sin 105^0 = \frac{1}{2}\sqrt{2 + \sqrt{3}}$; $\operatorname{tang} 105^0 = -(2 + \sqrt{3})$. b) $\frac{1}{p}$. c) q.

60. $\sin x = \pm\cos a$; $90^0 - a$, $90^0 + a$, $270^0 - a$, $270^0 + a$.

61. $\operatorname{tang} x = -1$; 135^0, 315^0. **62.** $\cos x = a$.

63. $\operatorname{tang} x = \pm\sqrt{b}$. **64.** $\operatorname{tang} x = c$.

65. $\sin x = \pm\sqrt{\frac{c-b}{a-b}}$. **66.** $\operatorname{tang} x = 1$; 45^0, 225^0.

67. $\sin x = \pm\sqrt{\frac{b}{a}}$.

68. $\sin x = 0$ oder $\cos x = \frac{b}{a} = \frac{1}{2}$; 0^0, 60^0, 180^0, 300^0.

69. a) $\sin x = 0$ und $\cos x = \frac{b-a}{b+a} = \frac{\sqrt{5}+1}{4}$; 0^0, 36^0, 180^0, 324^0.

b) $\cos x = 0$ und $\sin x = \frac{b-a}{b+a} = \frac{\sqrt{5}-1}{4}$; 18^0, 90^0, 162^0, 270^0.

70. a) $\cos x = -1$ u. $\operatorname{tang} x = \frac{b}{a} = 1$; 45^0, 180^0, 225^0.

b) $\cos x = 1$ und $\operatorname{tang} x = \frac{b}{a} = 1$; 0^0, 45^0, 225^0.

c) $\sin x = -1$ und $\operatorname{cotg} x = \frac{b}{a} = 1$; 45^0, 225^0, 270^0.

d) $\sin x = 1$ und $\operatorname{cotg} x = \frac{b}{a} = 1$; 45^0, 90^0, 225^0.

71. a) $\cos x = -1$; $\sin x = \frac{b}{a} = 1$; 90^0, 180^0.

b) $\cos x = 1$; $\sin x = \frac{b}{a} = 1$.

c) $\sin x = -1$, $\cos x = \frac{b}{a} = 1$.

d) $\sin x = 1$, $\cos x = \frac{b}{a} = 1$.

72. a) $\cos x = -1$, $\cos x = (b-a) : b = \frac{1}{2}$.
b) $\cos x = 1$, $\cos x = (a-b) : b = -\frac{1}{2}$.
c) $\sin x = -1$, $\sin x = (b-a) : b = \frac{1}{2}$.
d) $\sin x = 1$, $\sin x = (a-b) : b = -\frac{1}{2}$.

73. a) $\cos x = -1$, $\cos x = \frac{b}{a+b} = \frac{1}{4}$.

b) $\sin x = 1$, $\sin x = \frac{b}{a-b} = \frac{1}{2}$.

74. a) $\operatorname{tg} x = \frac{1}{2} a \pm \sqrt{\frac{1}{4} a^2 \mp 1}$.
b) $\sin x = \frac{a^2-1}{a^2+1}$ oder $\operatorname{tang} x = \frac{a^2-1}{2a}$.

75. $\sin x = \frac{-a+\sqrt{a^2+4b^2}}{2b} = \frac{1}{2}$.

76. $\cos x = \frac{-b+\sqrt{4a^2+b^2}}{2a} = \frac{\sqrt{5}-1}{2}$.

77. a) $\cos x = \frac{1}{2}(\sqrt{4+a^2} - a)$. b) $\sin x = \sqrt{\frac{a \pm \sqrt{a^2-4b^2}}{2a}}$.

78. $\cos x = \frac{1}{2}(1 \pm \sqrt{5-4a}) = 1$ oder 0.

79. $\sin x = \frac{c + \sqrt{c^2 \mp 4ab}}{2a} = 1$ oder $1 - \sqrt{2}$.

80. $\cos x = \frac{1}{8}(-1 \pm \sqrt{33})$.

81. $\cos x = \frac{-c \pm \sqrt{4a(a-b) + c^2}}{2(a-b)}$.

82. $\operatorname{tang} x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

83. $\operatorname{tang} x = \sqrt[3]{\frac{\sqrt{ab(a+2b)(b+2a)} \pm (a+b)\sqrt{2ab}}{a^2}}$.

84. $\operatorname{tang} x = \sqrt[3]{0{,}3}$. **85.** $\operatorname{tang} x = -\sqrt[3]{\frac{16}{17}}$.

86. $\cos x = \frac{1}{2}\sqrt{3}$ oder $\frac{1}{3}\sqrt{6}$.

87. a) $\operatorname{tang} y = -\frac{1}{2} \pm \sqrt{\frac{b}{a} + \frac{1}{4}}$; $\frac{\pm\sqrt{5} - 1}{2}$ oder $\frac{1}{2}\sqrt{\frac{7 + 4\sqrt{3}}{3}} - \frac{1}{2}$, d. i. $\frac{1}{3}\sqrt{3}$ und $-(1 + \frac{1}{3}\sqrt{3})$.

b) $\operatorname{cotg} y$ wie a.

c) $\operatorname{tang} y = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{b}{a}}$; $\frac{1}{2}$ oder $\sqrt{3} - 1$ u. $2 - \sqrt{3}$.

d) $\operatorname{cotg} y$ wie c.

88. a) $\operatorname{tang} y = \frac{b \pm \sqrt{b^2 - 4a^2}}{2a}$; 1 oder 2 und $\frac{1}{2}$.

b) $\operatorname{tang} y = \frac{-b \pm \sqrt{b^2 + 4a^2}}{2a} = \frac{1}{2}$ u. -2, od. $-1 \pm \sqrt{2}$.

89. a) $\operatorname{tang} y = \frac{c \pm \sqrt{c^2 - 4ab}}{2a} = 1$ oder $\frac{3}{2}$ und 1 oder $1 \pm \sqrt{4 - 2\sqrt{3}}$.

b) $\operatorname{tang} y = \frac{c \pm \sqrt{c^2 + 4ab}}{2a}$; $\frac{1 \pm \sqrt{5}}{2}$ oder $1 \pm \sqrt{2}$ oder $1 \pm \sqrt{\frac{4 - 2\sqrt{3}}{3}}$.

90. $\sin x = 0$, $\sin y = 0$ oder $\sin x = \sqrt{\frac{b^2 - a^2}{b^2 - 1}}$, $\sin y = \frac{1}{a}\sqrt{\frac{b^2 - a^2}{b^2 - 1}}$.

91. $\sin y = \sqrt[4]{0{,}8} = \operatorname{tang} x$.

92. $\sin x = \pm\sqrt{\frac{1}{2}(a - b)}$, $\cos y = \pm\sqrt{\frac{1}{2}(a + b)}$.

93. $\sin x = b\sqrt{\frac{c-1}{b^2-a^2}}$, $\cos y = a\sqrt{\frac{c-1}{b^2-a^2}}$.

94. $\operatorname{tg} x = \frac{4}{3}$, $\operatorname{tg} y = -8$ und $\operatorname{tg} x = \frac{2}{3}$, $\operatorname{tg} y = -2$.

95. $\cos x = \sqrt{\mp\sqrt{-b^2+\frac{1}{4}(a^2-b^2-1)^2}-\frac{1}{2}(a^2-b^2-1)}$,
$\sin y = \sqrt{\pm\sqrt{-b^2+\frac{1}{4}(a^2-b^2-1)^2}-\frac{1}{2}(a^2-b^2-1)}$.

96. $\sin x = \frac{\sqrt{5}\mp 1}{2}$, $y = 270^0 - x$.

97. $x = 90^0$, $y = 0$, $z = 0$ oder 180^0 und $x = 270^0$, $y = 180^0$, $z = 0$ oder 180^0.

98. $\operatorname{tang} x = \frac{1}{2}\left[\sqrt{(a-b)^2 + 4\cdot\frac{a-b}{c}} + a - b\right]$,

$\operatorname{tang} y = \frac{1}{2}\left[a + b - \sqrt{(a-b)^2 + 4\cdot\frac{a-b}{c}}\right]$,

$\operatorname{tang} z = \frac{1}{2}\left[\sqrt{(a-b)^2 + 4\cdot\frac{a-b}{c}} - a + b\right]$.

99. $x = 30^0, 150^0, 210^0, 330^0$;
$y = 30^0, 150^0, 210^0, 330^0$;
$z = 45^0, 135^0, 225^0, 315^0$.

100. $\sin x = \pm 8 : \sqrt{65}$,
$\sin y = \pm 4 : \sqrt{65}$,
$\sin z = \pm 2 : \sqrt{65}$,
$\sin u = \pm 1 : \sqrt{65}$.

§. 5.

1. a) $\sin(\alpha \pm \beta) = \pm\frac{1}{2}\sqrt{3}$; $\cos(\alpha \pm \beta) = \mp\frac{1}{2}$.
b) $\sin(\alpha + \beta) = \frac{56}{65}$, $\sin(\alpha - \beta) = \frac{16}{65}$; $\cos(\alpha + \beta) = \frac{33}{65}$, $\cos(\alpha - \beta) = \frac{63}{65}$.

2. a) $\operatorname{tang}(\alpha \pm \beta) = \frac{q \pm p}{pq \mp 1}$, $\frac{1}{2}$; $-\frac{2}{11}$.
$\operatorname{cotg}(\alpha \pm \beta) = \frac{pq \mp 1}{q \pm p}$, $+2$; $-\frac{11}{2}$.
b) $\operatorname{tang}(\alpha + \beta) = -10{,}2$, $\operatorname{tang}(\alpha - \beta) = -\frac{49}{15}$, $\operatorname{cotg}(\alpha + \beta) = -\frac{5}{51}$, $\operatorname{cotg}(\alpha - \beta) = -\frac{15}{49}$.

3. $\frac{\sec\alpha\cdot\sec\beta\cdot\operatorname{cosec}\alpha\cdot\operatorname{cosec}\beta}{\operatorname{cosec}\alpha\cdot\operatorname{cosec}\beta \mp \sec\alpha\cdot\sec\beta}$; $\frac{\sec\alpha\cdot\operatorname{cosec}\beta\cdot\operatorname{cosec}\alpha\cdot\sec\beta}{\sec\alpha\cdot\operatorname{cosec}\beta \pm \operatorname{cosec}\alpha\cdot\sec\beta}$.

4. a) $\sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\sin\gamma$.

b) $\cos\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\sin\gamma + \sin\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\cos\gamma$.

c) $\dfrac{\operatorname{tang}\alpha - \operatorname{tang}\beta + \operatorname{tang}\gamma + \operatorname{tang}\alpha \,.\, \operatorname{tang}\beta \,.\, \operatorname{tang}\gamma}{1 + \operatorname{tang}\alpha \,.\, \operatorname{tang}\beta - \operatorname{tang}\alpha \,.\, \operatorname{tang}\gamma + \operatorname{tang}\beta \,.\, \operatorname{tang}\gamma}$.

d) $\dfrac{\operatorname{cotg}\alpha \,.\, \operatorname{cotg}\beta \,.\, \operatorname{cotg}\gamma - \operatorname{cotg}\alpha + \operatorname{cotg}\beta + \operatorname{cotg}\gamma}{\operatorname{cotg}\gamma \,.\, \operatorname{cotg}\beta - \operatorname{cotg}\gamma \,.\, \operatorname{cotg}\alpha - \operatorname{cotg}\beta \,.\, \operatorname{cotg}\alpha - 1}$.

5. a) $\sin\alpha\cos\beta\cos\gamma\cos\delta + \cos\alpha\sin\beta\cos\gamma\cos\delta + \cos\alpha\cos\beta\sin\gamma\cos\delta + \cos\alpha\cos\beta\cos\gamma\sin\delta - \cos\alpha\sin\beta\sin\gamma\sin\delta - \sin\alpha\cos\beta\sin\gamma\sin\delta - \sin\alpha\sin\beta\cos\gamma\sin\delta - \sin\alpha\sin\beta\sin\gamma\cos\delta$.

b) $\sin\alpha\cos\beta\cos\gamma\cos\delta + \cos\alpha\sin\beta\cos\gamma\cos\delta + \cos\alpha\cos\beta\sin\gamma\cos\delta - \cos\alpha\cos\beta\cos\gamma\sin\delta + \cos\alpha\sin\beta\sin\gamma\sin\delta + \sin\alpha\cos\beta\sin\gamma\sin\delta + \sin\alpha\sin\beta\cos\gamma\sin\delta - \sin\alpha\sin\beta\sin\gamma\cos\delta$.

c) $\sin\alpha\cos\beta\cos\gamma\cos\delta + \cos\alpha\sin\beta\cos\gamma\cos\delta - \cos\alpha\cos\beta\sin\gamma\cos\delta - \cos\alpha\cos\beta\cos\gamma\sin\delta - \cos\alpha\sin\beta\sin\gamma\sin\delta - \sin\alpha\cos\beta\sin\gamma\sin\delta + \sin\alpha\sin\beta\cos\gamma\sin\delta + \sin\alpha\sin\beta\sin\gamma\cos\delta$.

d) $\sin\alpha\cos\beta\cos\gamma\cos\delta - \cos\alpha\sin\beta\cos\gamma\cos\delta - \cos\alpha\cos\beta\sin\gamma\cos\delta - \cos\alpha\cos\beta\cos\gamma\sin\delta + \cos\alpha\sin\beta\sin\gamma\sin\delta - \sin\alpha\cos\beta\sin\gamma\sin\delta - \sin\alpha\sin\beta\cos\gamma\sin\delta - \sin\alpha\sin\beta\sin\gamma\cos\delta$.

6. a) $\cos\alpha\cos\beta\cos\gamma\cos\delta - \sin\alpha\sin\beta\cos\gamma\cos\delta - \sin\alpha\cos\beta\sin\gamma\cos\delta - \sin\alpha\cos\beta\cos\gamma\sin\delta - \cos\alpha\sin\beta\sin\gamma\cos\delta - \cos\alpha\sin\beta\cos\gamma\sin\delta - \cos\alpha\cos\beta\sin\gamma\sin\delta + \sin\alpha\sin\beta\sin\gamma\sin\delta$.

b)

$$\frac{\operatorname{tg}\alpha - \operatorname{tg}\beta + \operatorname{tg}\gamma + \operatorname{tg}\delta + \operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma + \operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\delta - \operatorname{tg}\alpha\operatorname{tg}\gamma\operatorname{tg}\delta + \operatorname{tg}\beta\operatorname{tg}\gamma\operatorname{tg}\delta}{1 + \operatorname{tg}\alpha\operatorname{tg}\beta - \operatorname{tg}\alpha\operatorname{tg}\gamma - \operatorname{tg}\alpha\operatorname{tg}\delta + \operatorname{tg}\beta\operatorname{tg}\gamma + \operatorname{tg}\beta\operatorname{tg}\delta - \operatorname{tg}\gamma\operatorname{tg}\delta - \operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma\operatorname{tg}\delta},$$

c)

$$\frac{\cot\delta\cot\gamma - \cot\delta\cot\beta + \cot\delta\cot\alpha + \cot\gamma\cot\beta - \cot\gamma\cot\alpha + \cot\beta\cot\alpha + \cot\alpha\cot\beta\cot\gamma\cot\delta + 1}{\cot\delta\cot\gamma\cot\beta - \cot\delta\cot\gamma\cot\alpha + \cot\delta\cot\beta\cot\alpha - \cot\gamma\cot\beta\cot\alpha + \cot\delta - \cot\gamma + \cot\beta - \cot\alpha}.$$

d) $\cos\alpha\cos\beta\cos\gamma\cos\delta + \sin\alpha\sin\beta\cos\gamma\cos\delta + \sin\alpha\cos\beta\sin\gamma\cos\delta - \sin\alpha\cos\beta\cos\gamma\sin\delta - \cos\alpha\sin\beta\sin\gamma\cos\delta + \cos\alpha\sin\beta\cos\gamma\sin\delta + \cos\alpha\cos\beta\sin\gamma\sin\delta + \sin\alpha\sin\beta\sin\gamma\sin\delta$.

7. a) $\frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha . \operatorname{tg}\beta}$, b) $\frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha . \operatorname{tg}\beta}$, c) $\frac{\cot\alpha . \cot\beta - 1}{\cot\beta - \cot\alpha}$,
d) $\frac{\cot\alpha . \cot\beta + 1}{\cot\beta + \cot\alpha}$, e) $\frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{\operatorname{tg}\alpha + \operatorname{tg}\beta} = \frac{\cot\beta - \cot\alpha}{\cot\beta + \cot\alpha}$,
f) $\frac{1 + \operatorname{tg}\alpha . \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha . \operatorname{tg}\beta} = \frac{\cot\alpha . \cot\beta + 1}{\cot\alpha . \cot\beta - 1}$.

8. $\sin(90^0 + x) = \sin 90^0 . \cos x + \cos 90^0 . \sin x = \cos x$, u. s. w.

11. $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$.

13. a) $\cos(a + b) =$ etc. b) $\cos(a - b) =$ etc.

48. $\sin x = 1$, $x = 90^0$.

49. a) $\operatorname{tang} x = \frac{c - \sin a}{\cos a - b}$, b) $\operatorname{tg} x = \frac{n + \cos a}{m - \sin a}$.

50. $\operatorname{cotg} x = \sqrt{1 + 2 \operatorname{tg} a^2}$.

51. $\operatorname{cotg} x = -2 - \operatorname{cotg} \vartheta$. 52. $\operatorname{tang} x = -1$.

53. $\operatorname{tang} x = \sqrt{\frac{a-2}{a}}$. 54. $\operatorname{tang} x = \sqrt{\frac{1 - \cot\gamma}{1 - \sin\gamma}}$.

55. $\operatorname{tang} x = \operatorname{tang}\alpha . \operatorname{tang}\beta . \operatorname{tang}\gamma$.

56. $\operatorname{tang} x = 0$ oder $\operatorname{tang} x^2 = \operatorname{cotg}\mu^4 \, [2 + 3 \operatorname{tang}\mu^2 \pm 2\sqrt{1 + \operatorname{tang}\mu^6 + 3 \operatorname{tang}\mu^2 (1 + \operatorname{tang}\mu^2)}]$.

57. $\sin x^2 = \frac{1}{2} [\sin\gamma \sqrt{- 4a^2 - 4a \cos\gamma + \sin\gamma^2} - 2a\cos\gamma + \sin\gamma^2]$.

58. $x = 45^0, 165^0, 285^0, 225^0$.
$y = 15^0, 135^0, 135^0, 75^0$.

59. $\sqrt{p^2 q^2 + p^2 + q^2 + 1} = \omega$,
$\operatorname{tang} x = \frac{pq - 1 \pm \omega}{p + q}$, $\operatorname{tang} y = \frac{-pq - 1 \pm \omega}{p - q}$.

60. a) $\operatorname{tg} y = \frac{p \sin\delta - q \sin\varepsilon}{p\cos\delta - q\cos\varepsilon}$, $x = \frac{p}{\sin(\varepsilon - y)} = \frac{q}{\sin(\delta - y)}$.
b) $\operatorname{tang} y = \frac{p\cos\delta - q\cos\varepsilon}{q\sin\varepsilon - p\sin\delta}$, $x = \frac{p}{\cos(\varepsilon - y)} = \frac{q}{\cos(\delta - y)}$.

61. $\cos x = \pm 0{,}6$; $\cos y = \pm 0{,}8$.

§. 6.

1. $\sin 36^0 = \frac{1}{4}\sqrt{10 - 2\sqrt{5}}$, $\cos 36^0 = \frac{1}{4}(\sqrt{5} + 1)$, $\tan 36^0 = \sqrt{5 - 2\sqrt{5}}$.

2. $\sin 2x = 0{,}48050$, $\cos 2x = -0{,}87699$,
$\operatorname{tg} 2x = -0{,}54790$, $\cot 2x = -1{,}82515$.

3. a) $\cos x = \frac{1 - a^2}{1 + a^2}$; 0 oder 0,6 oder $\frac{1}{2}\sqrt{2}$.

b) $\sin x = \frac{2a}{1 + a^2}$; 1 oder $\frac{2}{3}\sqrt{2}$ oder $\frac{1}{2}\sqrt{2}$.

c) $\cos 2x = -\frac{1}{2}$, $\operatorname{tang} 2x = -\sqrt{3}$.

4. $\sin 3\alpha = 3 \sin\alpha \cos\alpha^2 - \sin\alpha^3$;
$\cos 3\alpha = \cos\alpha^3 - 3\sin\alpha^2 \cos\alpha$;
$\operatorname{tang} 3\alpha = \frac{3\operatorname{tang}\alpha - \operatorname{tang}\alpha^3}{1 - 3\operatorname{tang}\alpha^2}$; $\operatorname{cotg} 3\alpha = \frac{\operatorname{cotg}\alpha^3 - 3\operatorname{cotg} a}{3\operatorname{cotg}\alpha^2 - 1}$.

5. $\sin 4\alpha = 4\sin\alpha\cos\alpha^3 - 4\sin\alpha^3\cos\alpha$;
$\cos 5\alpha = \cos\alpha^5 - 10\sin\alpha^2\cos\alpha^3 + 5\sin\alpha^4\cos\alpha$.
$\operatorname{tang} 6\alpha = \frac{6\operatorname{tang}\alpha - 20\operatorname{tang}\alpha^3 + 6\operatorname{tang}\alpha^5}{1 - 15\operatorname{tang}\alpha^2 + 15\operatorname{tang}\alpha^4 - \operatorname{tang}\alpha^6}$.

6. $\sin 2\alpha = 2\sin\alpha\cos\alpha$.

7. $\cos 2\alpha = 1 - 2\sin\alpha^2 = 2\cos\alpha^2 - 1$.

48. $\sin 2x = 2a$.

49. $\sin 2x = 2b - \sin 2a$.

50. $\sin x = 0$, $\cos x = \frac{1}{4}(\sqrt{17} - 1)$.

51. a) $\sin x = 0$, $\cos x = \frac{b}{2a} = \frac{1}{2}$;

b) $\cos x = 0$, $\sin x = \frac{b}{2a} = \frac{1}{2}$.

52. a) $\sin 2x = \frac{a}{b - a} = \frac{1}{2}$; b) $\sin 2x = \frac{a}{b + a} = \frac{1}{2}$.

53. a) $\operatorname{tg} x = -1$, $\operatorname{tg} x = \frac{a}{2b + a} = \frac{1}{3}$ oder $2 - \sqrt{3}$;

b) $\operatorname{tg} x = 1$, $\operatorname{tg} x = \frac{a}{2b - a} = 1$ oder $-\sqrt{2} - 1$.

c) $\operatorname{tg} x = -1$, $\operatorname{tg} x = \frac{a - 2b}{a} = -1$; $\sqrt{2} - 1$.

d) $\operatorname{tg} x = 1$, $\operatorname{tg} x = \frac{2b - a}{a} = 1$; $2 + \sqrt{3}$.

54. a) $\sin 2x = -1$, $\sin 2x = \frac{4a^2 - b^2}{4a^2 + b^2} = 0{,}6$;

b) $\sin 2x = 1$, $\sin 2x = \frac{b^2 - 4a^2}{b^2 + 4a^2} = -0{,}6$.

55. a) $\cos 2x = -1$, $\cos 2x = \frac{b^2 - 4a^2}{b^2 + 4a^2} = -0{,}6$;

b) $\cos 2x = 1$, $\cos 2x = \frac{4a^2 - b^2}{4a^2 + b^2} = 0{,}6$.

56. a) $\cos 2x = 0$, $\sin 2x = \frac{2a}{b} = \frac{1}{2}$;

b) $\cos 2x = \frac{b}{2a} = \frac{1}{2}$; c) $\cos 2x = \frac{2a}{b} = \frac{1}{2}$.

57. a) $\operatorname{tang} x = 0{,}8$ oder $\frac{2}{7}$; b) $\operatorname{tang} x = 1$ oder -2.

58. a) $\sin x = 0$, $\cos x = \frac{d}{c}$; b) $\cos x = 0$, $\sin x = \frac{d}{c}$.

59. a) $\sin x = 0$, $\cos x = \frac{n}{2m}$; b) ebenso.

60. a) $\sin x = 0$, $\operatorname{tang} x = \pm 1$, $\sin 2x = \frac{b}{a}$;

b) $\sin x = 0$, $\sin 2x = n : m$.

61. a) $\sin x = 0$, $\cos x = 0$, $\operatorname{cotg} x = b$.

b) $\sin x = 0$, $\operatorname{tang} x = b : a$. c) $\operatorname{tang} x = n : m$.

62. a) $\sin 2x = \frac{a}{b-a}$; b) ebenso; c) $\sin 2x = \frac{m}{n-m}$;

d) $\sin 2x = -\frac{1}{2} \pm \sqrt{\frac{n}{m} + \frac{1}{4}}$.

63. a) $\operatorname{cotg} x = -1$, $\operatorname{cotg} x = 2\frac{a}{m} + 1$;

b) $\operatorname{tang} x = -1$, $\operatorname{cotg} x = \frac{a + 2b}{a}$;

c) $\operatorname{cotg} x = 1$, $\operatorname{cotg} x = \frac{2d - c}{c}$;

d) $\operatorname{cotg} x = 1$, $\operatorname{cotg} x = \frac{2n - m}{m}$;

e) $\operatorname{cotg} x = -1$, $\operatorname{cotg} x = \frac{a}{a - 2b}$;

f) $\operatorname{tg} x = -1$, $\operatorname{tg} x = \frac{a - 2d}{a}$; g) $\operatorname{cotg} x = \frac{m}{m - 2n}$,

64. $\operatorname{cotg} x = 1$; $\frac{a}{2b - a}$.

65. a) $\operatorname{cotg} 2x = \frac{4mn}{n^2 - 4m^2}$; b) $\operatorname{tg} x = \pm 1$, $\operatorname{tg} x = \frac{2a}{b}$;

c) $\operatorname{cotg} 2x = 0$, $\operatorname{tg} x = \frac{2m}{n}$;

d) $\operatorname{cotg} 2x = 0$, $\operatorname{cotg} x = 2a : b$;

e) $\operatorname{cotg} 2x = 0$, $\operatorname{cotg} x = 2m : n$;
f) $\operatorname{tang} x = \pm 1$, $\sin 2x = 2a : b$;
g) $\cos 2x = 0$, $\sin 2x = 2m : n$.

66. $x = 0$, $x = \pm\sqrt{2}$.

67. $\operatorname{tg} x = 0$ oder $\pm\sqrt{\frac{1}{8}(1+\sqrt{17})}$.

68. $\sin x = 0$ oder $\cos x = 0$ oder $\cos x = \frac{2}{3}$.

69. $\cos x = 0$ oder $\sin x = \pm\sqrt{\frac{1}{3}a}$.

70. $\sin x = 0$ oder $\operatorname{tg} x = 3$. **71.** $\operatorname{tang} x = -1$.

72. $\operatorname{tg} x = 0$ oder $\pm\sqrt{3}$. **73.** $\cos 2x = 0$.

74. $\cos(45^0 - 4x) = 0$ oder $\cos x = \frac{1}{4}\sqrt{2}$.

75. $\cos\frac{1}{2}x = 0$; $\sin\frac{1}{2}x = \frac{1}{4}$.

§. 7.

1. $\sin\frac{1}{2}\alpha = \frac{5}{13}$, $\cos\frac{1}{2}\alpha = \pm\frac{12}{13}$, $\operatorname{tang}\frac{1}{2}\alpha = \pm\frac{5}{12}$, $\operatorname{cotg}\frac{1}{2}\alpha = \pm\frac{12}{5}$.

2. a) $\sin\frac{1}{2}x = \sqrt{\frac{1-0{,}4\sqrt{6}}{2}} = 0{,}10051$;

$\cos\frac{1}{2}x = \sqrt{\frac{1+0{,}4\sqrt{6}}{2}} = 0{,}99494$.

b) $\frac{1}{7}(\sqrt{50}-1)$ oder $-\frac{1}{7}(\sqrt{50}+1)$.

3. $\sin 22\frac{1}{2}^0 = \frac{1}{2}\sqrt{2-\sqrt{2}} = 0{,}38268$;
$\cos 22\frac{1}{2}^0 = \frac{1}{2}\sqrt{2+\sqrt{2}} = 0{,}92388$;
$\operatorname{tang} 22\frac{1}{2}^0 = \sqrt{2}-1 = 0{,}41421$;
$\operatorname{cotg} 22\frac{1}{2}^0 = \sqrt{2}+1 = 2{,}41421$.

4. $\sin 15^0 = \frac{1}{2}\sqrt{2-\sqrt{3}} = \frac{1}{4}\sqrt{2}(\sqrt{3}-1)$;
$\cos 15^0 = \frac{1}{2}\sqrt{2+\sqrt{3}} = \frac{1}{4}\sqrt{2}(\sqrt{3}+1)$;
$\operatorname{tang} 15^0 = 2-\sqrt{3} = (3-\sqrt{3}) : (3+\sqrt{3})$;
$\operatorname{cotg} 15^0 = 2+\sqrt{3} = (\sqrt{3}+1) : (\sqrt{3}-1)$.

5. $\frac{\sqrt{1+\operatorname{tang} 2\alpha^2}-1}{\operatorname{tang} 2\alpha}$.

6. $\sin\frac{1}{3}\alpha^3 - \frac{3}{4}\sin\frac{1}{3}\alpha + \frac{1}{4}\sin\alpha = 0$,
$\cos\frac{1}{3}\alpha^3 - \frac{3}{4}\cos\frac{1}{3}\alpha - \frac{1}{4}\cos\alpha = 0$,
$\operatorname{tg}\frac{1}{3}\alpha^3 - 3\operatorname{tg}\alpha\operatorname{tg}\frac{1}{3}\alpha^2 - 3\operatorname{tg}\frac{1}{3}\alpha + \operatorname{tg}\alpha = 0$.

11. a) $\sin \frac{1}{2} x = 0$; $\sin \frac{1}{2} x = \frac{b}{2a} = \frac{1}{2}$;

b) $\cos \frac{1}{2} x = 0$; $\cos \frac{1}{2} x = \frac{b}{2a} = \frac{1}{2}$.

12. a) $\sin \frac{1}{2} x = 0$; $\sin x = \frac{b}{a} = 1$;

b) $\cos \frac{1}{2} x = 0$; $\sin x = \frac{b}{a} = 1$.

13. a) $\cos \frac{1}{2} x = 0$, $\operatorname{tang} \frac{1}{2} x = \frac{a}{b} = 1$;

b) $\sin x = 0$, $\operatorname{tang} x = \frac{b}{a} = 3$.

14. $\operatorname{tang} x = \frac{d}{c}$.

15. a) $\sin x = 0$, $\cos x = \frac{b}{a}$; b) $\cos x = 0$, $\sin x = \frac{b}{a}$.

16. a) $\sin x = 0$, $\cos x = \frac{b}{a}$; b) $\cos x = 0$, $\sin x = \frac{b}{a}$.

17. a) $\sin x = 0$, $\cos x = d : 2c$;

b) $\cos x = 0$, $\sin x = d : 2c$.

18. $\cos x = 0$, $\sin 2x = b : a$.

19. a) $\sin 2x = \frac{a}{a+b}$; b) ebenso.

§. 8.

1. a) $2 \sin 90^\circ . \cos 15^\circ = 2 \cos 15^\circ$;

b) $2 \cos 45^\circ . \cos 30^\circ = \frac{1}{2} \sqrt{6}$; c) $2 \sin 52^\circ . \cos 64^\circ$;

d) $2 \sin 32^\circ . \sin 21^\circ$; e) $- 2 \sin 90^\circ . \sin 45^\circ = - \sqrt{2}$;

f) $2 \sin 180^\circ . \cos 60^\circ = 0$.

2. a) $\frac{\sin 30^\circ . \cos 20^\circ}{\cos 54^\circ . \sin 20^\circ}$; b) $\frac{\cos 111^\circ . \cos 11^\circ}{\sin 74^\circ . \sin 32^\circ}$.

3. $\frac{\sin a + \sin b}{\sin a - \sin b} = \frac{\operatorname{tg} \frac{1}{2}(a+b)}{\operatorname{tg} \frac{1}{2}(a-b)}$, $\frac{\sin a + \sin b}{\cos a + \cos b} = \operatorname{tg} \frac{1}{2}(a+b)$,

$\frac{\sin a + \sin b}{\cos a - \cos b} = - \operatorname{cotg} \frac{1}{2}(a-b)$, $\frac{\sin a - \sin b}{\cos a + \cos b} = \operatorname{tg} \frac{1}{2}(a-b)$,

$\frac{\sin a - \sin b}{\cos a - \cos b} = - \operatorname{cotg} \frac{1}{2}(a+b)$, $\frac{\cos a + \cos b}{\cos a - \cos b} = - \operatorname{cotg} \frac{1}{2}(a+b) . \operatorname{cotg} \frac{1}{2}(a-b)$.

4. $4 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta$;

$4 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta + 1$.

14. a) $\cos x = 0$, $\sin 3x = b : 2a = \frac{1}{2}$,
$x = 10^0, 50^0, 130^0, 170^0$, u. s. w.
b) $\sin x = 0$, $\cos 3x = b : 2a = \frac{1}{2}$,
$x = 20^0, 100^0$, u. s. w.
c) $\cos x = 0$, $\cos 3x = b : 2a = \frac{1}{2}$,
d) $\sin x = 0$, $\sin 3x = - b : 2a = - \frac{1}{2}$.

15. a) $\cos(x - 45^0) = \frac{1}{2} a \sqrt{2} = 1$,
bezw. $\sin(x - 45^0) = \frac{1}{2} a \sqrt{2} = 1$.
b) $\cos(x - 45^0) = \frac{1}{2} b : \sin(45^0 + a)$.

16. $\cos x = 0$, $\cos(n-1)x = \frac{1}{2}$.

17. $\cos x = \frac{1}{4}(1 \pm \sqrt{5})$.

18. $\cos x = 0$ oder $\cos 4x = \frac{1}{2}$.

19. a) $x - y = 10^0$; $x = 50^0$, $y = 40^0$. b) 50^0; 30^0. c) 31^0; 11^0. d) 76^0; 6^0. e) 76^0; 6^0. f) 31^0; 11^0. g) $42^0 40'$; $77^0 10'$. h) $66^0 30'$; $43^0 30'$.

Die sonstigen Resultate der Zahlenbeispiele sind der Kürze halber hier und im Folgenden weggelassen; sie ergeben sich eventuell leicht aus den vorstehenden.

20. a) $43^0 30'$; $66^0 30'$. b) 67^0; 23^0. c) $5^0 26'$; $88^0 34'$. d) $17^0 0',9$; $23^0 10',9$.

21. a) 90^0; 30^0. b) 45^0; 30^0. c) 74^0; 60^0. d) 30^0; 0^0 oder 90^0; 60^0.

22. a) $23^0 47',61$; $126^0 12',39$. b) 20^0; 0^0. c) 23^0; 90^0.

23. a) $42^0 30'$; $22^0 10'$. b) $21^0 45'$; $15^0 45'$. c) $17^0 40'$; $45^0 0'$. d) $52^0 46',4$; $0^0 46',4$.

Anhang 1.

1. $90^0 - a$. **2.** $1 : m$. **3.** 9^0.

4. $\frac{\text{tang } 33^0 \, . \, \text{cotg } 25^0}{\text{cotg } 41^0 \, . \, \text{tang } 44^0}$.
$\frac{\sin 12^0 \, . \, \text{tang } 9^0}{\text{tang } 6^0 \, . \, \sin 15^0} + \frac{\cos 39^0 \, . \, \sin 27^0 \, . \, \text{cotg } 39^0}{\sin 4^0 \, . \, \text{cotg } 12^0 \, . \, \sin 8^0}$.

5. $x = 1$, $y = \text{cotg } 44^0$. **6.** $\text{tang } \alpha^2 \, . \, \text{tang } \beta^2$; $(\text{tang } 43^0 \, . \, \text{cotg } 34^0)^2$.

7. 45^0, 225^0. **8.** $\sqrt{\frac{1}{5}(5 + 2\sqrt{5})}$. **9.** $2p : (1 - p^2)$.

10. $- 1,61812$. **11.** $x = 60^0$ oder 180^0, $y = 0^0$ oder 120^0.

12. 90^0, 270^0. **15.** Wenn einer der Winkel α, β, $\alpha + \beta$ gleich 0 oder 360^0 ist.

16. $\frac{(n-m)^2}{\sin 2\alpha^2} + \frac{(m+n-\cos\varphi)^2}{\cos 2\alpha^2} = 1.$

17. $B + \sqrt{-AB} = 2\sqrt{-\frac{B}{A+B}}.$

18. $2 + m^2 + n^2 = \pm m\sqrt{4+m^2} \pm n\sqrt{4+n^2}.$

66. a) und b) $\sin x = \frac{\sqrt{5}-1}{2}.$

67. a) $\sin x = 0$, $\cos x = \frac{b}{a} = \frac{1}{2}$;

b) $\cos x = 0$, $\sin x = \frac{b}{a} = \frac{1}{2}$.

68. $\cos x = -1$ oder $\frac{1}{2}$.

69. $\sin x = \frac{3}{5}$. **70.** $\operatorname{tang} x = \pm 1$.

71. $\operatorname{tang} x = -\operatorname{tang}\beta$ oder $\operatorname{cotg} x = 1 - \operatorname{cotg}\beta$.

72. a) $\cos(45^0 - x) = \frac{b\sqrt{2}}{2a} = \frac{1}{2}\sqrt{2}$, $x = 0$, 90^0;

b) $\sin(45^0 - x) = \frac{b\sqrt{2}}{2a} = \frac{1}{2}\sqrt{2}$; 0, 270^0.

73. a) $\operatorname{tg} x = -1$, $\sin(45^0 - x) = \frac{a\sqrt{2}}{2b} = \frac{1}{2}\sqrt{2}$;

b) $\operatorname{tg} x = 1$, $\cos(45^0 - x) = \frac{a\sqrt{2}}{2b} = \frac{1}{2}\sqrt{2}$.

c) $\operatorname{tg} x = -1$; $\sin(45^0 - x) = \frac{b\sqrt{2}}{2a} = \frac{1}{2}\sqrt{2}$;

d) $\operatorname{tg} x = 1$, $\cos(45^0 - x) = \frac{b\sqrt{2}}{2a} = \frac{1}{2}\sqrt{2}$.

74. a) $\operatorname{tg} x = -1$; $\cos(45^0 - x) = \frac{a\sqrt{2}}{2b} = \frac{1}{2}\sqrt{2}$;

b) $\operatorname{tg} x = 1$; $\sin(45^0 - x) = \frac{a\sqrt{2}}{2b} = \frac{1}{2}\sqrt{2}$.

75. a) $\operatorname{tg} x = -1$, $\operatorname{tg} x = \frac{b-a}{b+a}$; b) $\operatorname{tg} x = 1$, $\operatorname{tg} x = \frac{a-b}{a+b}$.

76. a) $\operatorname{cotg} 2x = 0$, $\sin 2x = -\frac{b+2a}{2a}$;

b) $\cos 2x = 0$, $\sin 2x = \frac{2a}{2a+b}$.

77. a) $\operatorname{tang} 2x = 2a : b = 2$ oder 1;
b) $\sin 4x = 4a : b = 1$ oder $\frac{1}{2}$.

78. $\cos x = \frac{1}{2}\sqrt{3}$. **79.** $\operatorname{tg} x = \sqrt{3 \pm \sqrt{8}}$.

80. $\sin 3x = 0$ oder $\cos 3x = \pm \sqrt{\frac{1}{2}}$, $x = 15^0$, 45^0, 75^0, 105^0, 135^0, 165^0.

81. $\operatorname{tg} x = 0$ oder $\pm \operatorname{tg} \alpha$. **82.** $\operatorname{tg} x = \frac{\sin a \,.\, \sin b}{1 - \sin a \,.\, \cos b}$.

83. $\operatorname{tg} x = -\frac{1}{3}$. **84.** $\operatorname{tg} x = 1$ oder -2.

85. $\cos x = \frac{\sqrt{5}-1}{2}$. **86.** $x = 0$ oder $\operatorname{tg} x = -1$, $x = 135^0$, 315^0.

87. $\sin y = \sin x = 0$; $\cos x = \sqrt{1 - m^2} : \sqrt{1 - n^2}$, $\cos y = n\sqrt{1 - m^2} : m\sqrt{1 - n^2}$.

88. $\operatorname{tg} \frac{1}{2}(x + y) = \frac{a}{b}$, $\cos \frac{1}{2}(x - y) = \frac{a}{2 \sin \frac{1}{2}(x + y)}$, d. i. $\cos x = \frac{b \mp a w}{2}$, $\cos y = \frac{b \pm a w}{2}$, $w = \sqrt{(4 - a^2 - b^2) : (a^2 + b^2)}$.

89. $\sin \frac{1}{2} y = 0$, $\cos \frac{1}{2} y = \sqrt{\frac{-a \pm \sqrt{a^2 + 3}}{6a}}$.

90. $\sin x = 0$, $\operatorname{tang} y = 0$.

91. $x = 45^0, y = 30^0, z = 105^0$ od. $x = 135^0, y = 30^0, z = 15^0$.

92. $x = \sin \frac{1}{2}\alpha$ oder $\cos \frac{1}{2}\alpha$. **93.** $x = 0$, $x = \pm \sqrt{2}$.

94. $x = \sqrt{2} \,.\, \operatorname{cotg} 15^0$. **95.** $x = \pm ab$.

98. $x = 2a$. **99.** $\cos x = \sin 2a^2 : \cos 2a$.

§. 9.

1. $\sin 15^0 = \frac{1}{4}(\sqrt{6} - \sqrt{2})$, $\cos 15^0 = \frac{1}{4}(\sqrt{6} + \sqrt{2})$, $\operatorname{tang} 15^0 = 2 - \sqrt{3}$, $\operatorname{cotg} 15^0 = 2 + \sqrt{3}$.

2. $\sin 15^0 = \frac{1}{2}\sqrt{2 - \sqrt{3}} = 0{,}258819$,
$\cos 15^0 = \frac{1}{2}\sqrt{2 + \sqrt{3}} = 0{,}965926$.

3. $\sin 18^0 = 0{,}3090$, $\sin 36^0 = 0{,}5878$, $\cos 18^0 = 0{,}9511$.

4. $\sin 3^0 = \frac{1}{4}\sqrt{8 - \sqrt{3} - \sqrt{15} - \sqrt{10 - 2\sqrt{5}}}$.

5. $2 \sin 3^0 \,.\, \sqrt{1 - \sin 3^{0^2}} = 0{,}10453$.

6. $\sin \alpha = \frac{1}{2}\sqrt[3]{-a + \sqrt{a^2 - 1}} + \frac{1}{2}\sqrt[3]{-a - \sqrt{a^2 - 1}}$. Da $a < 1$, so ist $a^2 - 1$ negativ, also liegt der irreducibele Fall vor. Vergl. §. 7, 6.

7. 0,0029089. **8.** a) 0,0002909, b) 0,0000048.

9. Die Reihe muss convergiren. Dies ist stets der Fall, wenn cotg $3\alpha < 1$, d. i. $\alpha < 15^0$ ist. Der Ausdruck repräsentirt nur eine der drei reellen Wurzeln der Gleichung in 6, $\sin\alpha$, $-\sin(60^0-\alpha)$, $-\sin(60^0+\alpha)$. Die Berechnung für $\alpha=20^0$ giebt $-0,98481$, und da $\sin 60^0 = 0,86603$, so muss dies der Werth von $-\sin(60^0+\alpha)$, also $0,98481 = \cos 10^0$ sein.

14. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots;$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$

15. $\operatorname{tang} x = x + \frac{x^3}{1.3} + \frac{2x^5}{1.3.5} + \frac{17x^7}{1.3.5.7.3} + \ldots;$

$\operatorname{cotg} x = \frac{1}{x} - \frac{x}{1.3} - \frac{x^3}{1.3.5.3} - \frac{2x^5}{1.3.5.7.9} - \ldots$

16. a) 0,052; b) 0,914; c) 0,213.

18. $x = \sin x + \frac{1.\sin x^3}{2.3} + \frac{1.3.\sin x^5}{2.4.5} + \frac{1.3.5.\sin x^7}{2.4.6.7} + \ldots$

$x = \operatorname{tang} x - \frac{1}{3}\operatorname{tang} x^3 + \frac{1}{5}\operatorname{tang} x^5 - \frac{1}{7}\operatorname{tang} x^7 + \ldots$

19. $x = 0,197396$; $\alpha = 11^0\,18'\,36''$.

20. a) $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots;$

b) $\frac{\pi}{6} = \frac{1}{3}\sqrt{3}\left(1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \ldots\right);$

c) $\frac{\pi}{10} = \sqrt{\frac{5-2\sqrt{5}}{5}}\left(1 - \frac{1}{3}.\frac{5-2\sqrt{5}}{5} + \frac{1}{5}\left(\frac{5-2\sqrt{5}}{5}\right)^2 \ldots\right).$

§. 10.

132. 193,231131. **133.** 1,0016 mal. **134.** 55 Meilen. **135.** $0,00486\sqrt{l}$. **136.** 35844^m. **140.** $\frac{\pi}{4}$; $0,14189\,r$. **141.** 0,29375. **142.** $36^0\,35'\,12''$ und $23^0\,24'\,48''$.

Anhang 2.

1. $\sin 1 = \sin 57^0\,17'\,44'',8 = 0,84147$.

2. a) $\sin 1 - \cos 1 = 0,30116$; b) $\sin 1 + \cos 1 = 1,38178$.

3. Arc tg $(=\frac{1}{3}) = 0,32175$.

4. Arc tg $(=1) \pm$ Arc tg $(=\sqrt{\frac{1}{5}}) = 1,20593$; 0,36487.

5. a) $-\frac{x^3}{3!} + \frac{2x^5}{5!} - \frac{3x^7}{7!} + \ldots;$

b) $x - \frac{2x^3}{3!} + \frac{3x^5}{5!} - \frac{4x^7}{7!} + \ldots$

6. $1 - \frac{x^2 + xy + y^2}{3} + \frac{x^4 + x^3y + x^2y^2 + xy^3 + y^4}{5} - \ldots$

7. $\frac{1}{2} \operatorname{cotg} \frac{1}{2}x - \frac{\cos\frac{1}{2}(2n+1)x}{2\sin\frac{1}{2}x}$.

8. $-\frac{1}{2} + \frac{\sin\frac{1}{2}(2n+1)x}{2\sin\frac{1}{2}x}$. 9. $\frac{\sin nx^2}{\sin x}$.

10. $-\frac{1}{2} + \frac{\cos nx \,.\, \cos(n+1)y - \cos(n+1)x \,.\, \cos ny}{2(\cos y - \cos x)}$.

11. $\sin nx = \binom{n}{1} \cos x^{n-1} \sin x - \binom{n}{3} \cos x^{n-3} \sin x^3$
$+ \binom{n}{5} \cos x^{n-5} \sin x^5 - \ldots$

12. $\cos nx = \cos x^n - \binom{n}{2} \cos x^{n-2} \sin x^2 +$
$\binom{n}{4} \cos x^{n-4} \sin x^4 - \ldots$

13. $\operatorname{tg} nx = \frac{n \operatorname{tg} x - \binom{n}{3} \operatorname{tg} x^3 + \ldots \pm n \operatorname{tg} x^{n-1}}{1 - \binom{n}{2} \operatorname{tg} x^2 + \ldots \pm \operatorname{tg} x^n}$ für gerades n,

$\operatorname{tg} nx = \frac{n \operatorname{tg} x - \binom{n}{3} \operatorname{tg} x^3 + \ldots \pm \operatorname{tg} x^n}{1 - \binom{n}{2} \operatorname{tg} x^2 + \ldots \pm n \operatorname{tg} x^{n-1}}$ für ungerades n.

Anhang 3.

	φ	x_1	x_2
1.	42°	+ 0,69127	— 4,69127
2.	67° 8′	360,32	818,32
3.	55° 24′	136,61	495,61
4.	10° 0′	25,284	3303,3
5.	5° 40′	0,315	129,315
6.	37° 42′	72,04	618,04
7.	78° 27′ 47″	0,2	0,3
8.	84° 29′ 50″,9	99	120
9.	34° 30′ 0″,0	— 1035,2	+ 10737,25
10.	29. 28.	2,9715	42,971
11.	31. 23.	1,7136	21,7136
12.	31. 28.	4,3102	54,3102
13.	84. 5. 25,4	0,43563	0,53563
14.	18. 32. 57,2	0,2	7,5
15.	66. 25. 19.	3	7
16.	8. 5. 22,4	0,02	4

	φ	x_1	x_2
17.	21° 6′ 0″,0	— 1,6762	—48,3238
18.	31. 47. 18,2	0,3	3,7
19.	43.	1,34325	8,65675
20.	52.	96,081	403,919
21.	25. 56. 30,2	+ 0,6045	+11,3955
22.	58. 39.	239,87	760,13
23.	67. 40.	2,4801	5,5199
24.	10. 40. 49,3	0,1619	18,5296
25.	68° 0′ 13″,2	$-5 \pm 12{,}377\,i$	
26.	65. 12. 35,6	$-5 \pm 10{,}826\,i$	
27.	66. 31. 45,4	$2{,}5 \pm 5{,}758\,i$	
28.	68. 25. 54,3	$1 \pm 2{,}530\,i$	

	x_1	x_2		x_1	x_2
29.	— 12	— 13	37.	25,1	0,8
30.	— 0,9768	— 1,2345	38.	0,01	— 9,87
31.	— 1	— 0,5	39.	7,5	— 0,2
32.	+ 0,588285	+ 0,588285	40.	± 3	$\pm\sqrt{-14}$
33.	2	1	41.	4	1
34.	5	4	42.	2	— 3
35.	2	— 6	43.	306,8415	4,1585
36.	— 5	— 6			

51. $\sin 2\varphi = 2\sqrt{b} : a$, $\varphi = 37^0\,40'$, $x = 12$, $y = 20{,}137$. Für imaginäre Wurzeln wäre $\cos\varphi = a : 2\sqrt{b}$ zu setzen.

52. $\operatorname{tang} 2\varphi = 2\sqrt{b} : a$, $\varphi = 12^0\,20'$, $x = -0{,}28903$, $y = 5$.

53. $\sin 2\varphi = 2n : m$, $6^0\,20'\,24'',6$; 1,1; 9,9.

54. $\cos(45^0 - \varphi) = n : \sqrt{2m}$, $40^0\,14'\,11''$; 1,1; 1,3.

55. $\cos 2\varphi = n : m$; $x = \sqrt{p\,.\operatorname{cotg}\varphi}$; $\varphi = 36^0\,52'\,11'',6$; $x = \pm 6$, $y = \pm 8$.

56. $\operatorname{cotg} \frac{1}{2}\varphi = \sqrt{a : b}$, $x = m : \sin\varphi$; $53^0\,7'\,48'',7$; 30; 18.

57. $\operatorname{cotg} 2\varphi = -a : 2\sqrt{b}$, $x = \sqrt{b}\,.\operatorname{tang}\varphi$, $y = \sqrt{b}\,.\operatorname{cotg}\varphi$.

58. $\sin(\varphi - 45^0) = n : \sqrt{2m}$, $x = \sqrt{m}\sin\varphi$, $y = \sqrt{m}\,.\cos\varphi$.

59. $\operatorname{cotg} \frac{1}{2}\varphi = \sqrt{a : b}$, $x = m : \sqrt{1 + \cos\varphi^2}$, $y = x \,.\, \cos\varphi$, oder $\operatorname{cotg}(45^0 - \varphi) = a : b$, $x = m \cos\varphi$, $y = m \sin\varphi$.

60. $\sin 2\varphi = \frac{2q}{nb^2 - aq}$, $x = \sqrt{\frac{q}{n} \operatorname{tg}\varphi}$, $y = \sqrt{\frac{q}{n} \operatorname{cotg}\varphi}$.

61. $\operatorname{tg}(\varphi - \psi) = a$, $\sin(\varphi + \psi) = b$, $x = \operatorname{tg}\varphi$, $y \operatorname{tg}\psi$.

62. $\sin 2\varphi = -2\sqrt{b - 2} : a$, $\sin\psi = -\sqrt{4 : (b - 2)} \operatorname{tg}\varphi$, $\sin\psi' = -\sqrt{4 : (b - 2)} \operatorname{cotg}\varphi$, $x' = -\operatorname{tg}\frac{1}{2}\psi$, $x'' = -\operatorname{cotg}\frac{1}{2}\psi$, $x''' = -\operatorname{tg}\frac{1}{2}\psi'$, $x'''' = -\operatorname{cotg}\frac{1}{2}\psi'$. Für imaginäre Wurzeln $\cos\psi = \sqrt{\frac{1}{4}(b - 2)} \operatorname{tg}\varphi$ oder $\sqrt{\frac{1}{4}(b - 2)} \operatorname{cotg}\varphi$.
Oder $\operatorname{tang} 2\varphi = -\frac{2\sqrt{2 - b}}{a}$, $\sin\psi = \sqrt{\frac{4}{2 - b}} \operatorname{cotg}\varphi$, $\sin\psi' = -\sqrt{\frac{4}{2 - b}} \operatorname{tg}\varphi$, x wie vorher. Für imaginäre Wurzeln $\cos\psi = -\frac{1}{2}\sqrt{2 - b} \operatorname{tg}\varphi$ und $\cos\psi = +\frac{1}{2}\sqrt{2 - b} \operatorname{cotg}\varphi$, $x = \cos\psi \pm i \sin\psi$.

	φ	ψ	ψ'	x'	x''	x'''	x''''
α)	$51^0\,40'\,14'',0$	$30^0\;0'\;2'',7$	$306^0\,52'\,17'',6$	$+2$	$+0,5$	$-0,26795$	$-3,73205$
β)	17. 32. 54,5	36. 52. 11,4	80. 24. 21,1	$+3$	$-0,3333$	0,1667	$\pm 0,9860\,i$
γ)	54. 28. 22,6	11. 18. 27,6	247. 22. 48,0	$+5$	$+0,2$	$-10,101$	$-0,099$
δ)	54. 28. 22,6	191. 44. 6,9	247. 22. 37,5	$+9,7306$	$+0,1028$	$+1,5$	$+0,66667$
ε)	26. 19. 30	25. 1. 33,5	212. 3. 36,9	$+9,5352$	$+0,1048$	$+1,8050$	$+0,554.$
ζ)	33. 33. 30	253. 45. 0		$0,4583 \pm$	$0,8889\,i$		

63. $x = \pm 1$ und $x^4 + ax^3 + (b + 1)x^2 + ax + 1 = 0$. Vergl. 62.

Die reellen Wurzeln der Aufgaben 64—110 sind:

64. 15. **65.** 6. **66.** 7. **67.** 7. **68.** — 11. **69.** — 4. **70.** 4,0777. **71.** 4,3847. **72.** 9. **73.** — 144. **74.** 1,0634 *m*. **75.** — 7,6825. **76.** — 6,0221. **77.** 3. **78.** 10. **79.** 21. **80.** — 7. **81.** 7. **82.** — 9,4890. **83.** — 10,7343. **84.** — 11,8273. **85.** 5. **86.** 12. **87.** 3,5. **88.** — 1; — 2; + 3. **89.** — 5; + 7; — 2. **90.** — 9; + 6; + 3. **91.** — 12; + 8; + 4. **92.** + 22; — 11; — 11. **93.** — 0,4; + 0,3; + 0,1. **94.** — 1,234; — 5,678; + 6,912. **95.** 0,97; 3; — 3,97. **96.** 2; 1,45; — 3,45. **97.** — 2; 3,4495; — 1,4495.

98. 0,34730; 1,53208; —1,87938. **99.** $\sin 3\varepsilon = a$, $x_1 = \sin \varepsilon$. **100.** 8,2. **101.** —4. **102.** 5. **103.** 8. **104.** 13; 13; 13. **105.** 2; 4; 6. **106.** —7; —5; —3. **107.** 9; —2; +3. **108.** 11; —7; —6. **109.** 2,3; 4,6; 5,9. **110.** 0,003; —1,468; —214.

111. $\operatorname{tg}\varphi = \sqrt{b:a}$, $x = \sqrt{a} : \cos\varphi$.

112. $\cos\varphi = \sqrt{b:a}$, $x = \sqrt{a} \,.\, \sin\varphi$.

113. $\operatorname{tg}\varphi = \sqrt{b:a}$, $\cos\psi = \sqrt{d:c}$,
$\sin\vartheta = \sqrt{c:a} \,.\, \cos\varphi \,.\, \sin\psi$, $x = a\cos\vartheta^2 : \cos\varphi^2$.

114. $\operatorname{tg}\varphi = \sqrt{b:a}$, $x = \cos 2\varphi$, oder $\operatorname{tg}\psi = b:a$,
$x = \operatorname{tg}(45^0 - \psi)$.

115. $\operatorname{cotg}\varphi = a$, $x = \cos 2\varphi$.

116. $\operatorname{tg}\varphi = \sqrt{b:a}$, $\operatorname{tg}\psi = \sqrt{\cos 2\varphi}$, $x = 2a\cos(45^0 - \varphi)$.

117. $\cos\varphi = \sqrt{b:a}^{n}$, $x = \sqrt[m]{a^n \sin\varphi^2}$.

118. $\operatorname{tg}\varphi = b:a$, $x = a:\cos\varphi$.

119. $\cos\varphi = 2\cos\alpha\sqrt{ab} : (a+b)$, $x = (a+b)\sin\varphi$.

120. $\operatorname{cotg}(45^0 - \varphi) = \frac{a\sin\alpha}{b\sin\beta}$, $x = \frac{a\sqrt{2}\sin\alpha}{\cos(45^0-\varphi)}$.

121. $\operatorname{cotg}(\varphi - 45^0) = \frac{a\cos\alpha}{b\cos\beta}$, $x = \frac{a\sqrt{2}\cos\alpha}{\cos(\varphi - 45^0)}$.

122. $\cos 2\varphi = b\operatorname{tg}\beta : a\operatorname{tg}\alpha$,
$x = b \,.\, \operatorname{tg}\beta \,.\, \operatorname{tg} 2\varphi = a \,.\, \operatorname{tg}\alpha \,.\, \sin 2\varphi$.

123. $\operatorname{tg}\varphi = \frac{a}{b}$, $x = \frac{a}{\sin\varphi}\sin(\varphi + \frac{1}{2}\alpha)\sqrt{2\operatorname{tg}\alpha}$.

127. 42° 20′ 47″,2. **128.** 108° 36′ 13″,8; 1,6242r.
129. 66° 10′ 23″,5. **130.** 132° 20′ 47″. **131.** 84° 53′ 38″,8.
132. 149° 16′ 27″. **133.** 66° 46′ 54″,3.
134. 45° 54′ 52″,6. **135.** 40° 12′ 10″. **136.** 69° 30′ 2″,1.
137. 66° 46′ 54″,3. **138.** 54° 22′ 18″,7.

148. $\sqrt[n]{-\frac{1}{2}p \pm \sqrt{\frac{1}{4}p^2 - q}} \,.\, \sqrt[n]{1}$.

§. 13.

1. $2\sin\frac{1}{2}\alpha$; a) 0,73355; b) 1,21992; c) 1,76844.

2. $2r\sin\frac{1}{2}\alpha$; a) 2; b) 28,784; c) 4.

3. $\sin \frac{1}{2}\alpha = \frac{1}{2}s$; a) 44^0; b) 124^0.

4. $\sin \frac{1}{2}\alpha = s : 2r$; 67^0. **5.** $2r = \frac{d}{\sqrt{2} \sin (45^0 - \frac{1}{2}\gamma)}$.

6. 1 : 1,0107. **7.** $38^0\, 56'\, 33'',3$. **8.** 0,95885 : 1.

9. 10,252. **10.** $s : 2 \sin \alpha = 6{,}2494$. **11.** 64,349.

12. 2,580 oder 397,427. **13.** b) 15,569; 4285,231.

14. a) 120^0; b) $61^0\, 26'\, 37'',3$; $28^0\, 6'\, 37'',3$.

15. 2,8148; 3,90615; 2,25363.

16. a) 10; 36. b) 1034,15; 3962,9.

17. $\sqrt{2} - 1 : 1 = 0{,}41421 : 1$.

18. $\frac{P}{2\pi} \cos \frac{m \cdot 180^0}{m+n} = 15{,}279$.

19. a) $89^0\, 45'$; b) $89^0\, 52'\, 30''$.

§. 14.

1. 8. **2.** $\sqrt{\frac{4}{7} F \cdot \operatorname{tg} 25^0\, 42'\, 51'',4} = 3{,}45$.

3. $\sqrt{\frac{2a}{9 \cdot \sin 40^0}} = 10$. **4.** $\frac{1}{72} u^2 \cdot \operatorname{cotg} 10^0 = 835$.

5. $\frac{11}{5}\, a \cdot \frac{\sin 32^0\, 43'\, 38'',2}{\sin 72^0} = 4{,}15$.

6. $\frac{10 \cdot \operatorname{cotg} 20^0}{9 \cdot \operatorname{cotg} 18^0} = 0{,}9919 : 1$.

7. $\sqrt{a : (5 \cdot \operatorname{cotg} 9^0 - \frac{100}{11} \operatorname{cotg} (\frac{180}{11})^0)} = 22$.

8. $18\varrho^2 \operatorname{tg} 10^0 = 76{,}6$. **9.** $17 r^2 \operatorname{tg} \alpha \sin \alpha^2 = 0{,}4$.

10. 2 : 3.

§. 15.

1. $\frac{1}{2} d^2 \cdot \sin 2\alpha = 41274$.

2. $51^0\, 52'\, 43'',7$; $38^0\, 7'\, 16'',3$.

3. $\operatorname{tg} \frac{1}{2}\alpha = d : d_1$; 10^0; 170^0. **4.** $b = F : a \sin \alpha$; 83.

5. $\operatorname{tg} \frac{1}{2}\alpha = a : b$; 32^0; 148^0. **6.** $\sin \frac{1}{2}\alpha = r : a$; 42^0.

7. $\sin \frac{1}{2}\alpha = (R - r) : d$; 58^0. **8.** $\sqrt{2} : 1$.

9. 1472157 : 2160000.

10. $\operatorname{tg} \alpha_1 = \frac{b}{a}$, $\operatorname{tg} \alpha_2 = \frac{ab}{a^2 + 2b^2}$, $\operatorname{tg} \alpha_3 = \frac{ab}{a^2 + 6b^2}$;
162 : 66,3657 : 29,2682.

11. $\frac{1}{2} d^2 \sin\alpha = 129{,}8$. **12.** $\operatorname{tg}\alpha = 2\sqrt{2}$; $70^0\ 31'\ 44''$.

13. $\sin\frac{1}{2}\alpha = \frac{1}{2}\sqrt{\frac{n}{m}}$; $\alpha = 68^0$. **14.** $1 : 2\sin\frac{1}{2}\beta$; $7 : 1$.

15. 149. **16.** $\operatorname{tg}\alpha = 4F : (a^2 - b^2)$; $\alpha = 41^0$.

17. $d = c\sin\alpha : \sin\beta = 14$, $b = 25$.

18. $\alpha = 63^0\ 30'$, $\beta = 26^0\ 30'$.

19. $x = 2a\operatorname{tg}\frac{1}{2}\beta \,.\, \operatorname{tg}(45^0 - \frac{1}{4}\beta) = 15{,}396$.

20. 130^0. **21.** $\frac{c^2}{4}(\pi - 2)$. **22.** 60,153; — 5,1158.

23. $36^0\ 52'\ 11'',6$; $53^0\ 7'\ 48'',4$.

24. $\operatorname{tg}\varphi = \pm\frac{n+m}{n-m}$; 77^0, 103^0. **26.** a) 4,8741, b) 4,09.

27. $\cos\frac{1}{2}\alpha : \cos\frac{1}{6}\alpha$; $1 : 1{,}87938$.

28. $a \,.\, \operatorname{tg}\alpha$; a) 71; b) 35,3; c) $33\frac{1}{3}$.

29. $b \,.\, \operatorname{tg}\alpha - a = 179{,}5$. **30.** $a \,.\, \operatorname{tg}\beta = 20$.

31. $a \,.\, \operatorname{cotg}\alpha = 364$; 365.

32. $(a - b) \,.\, \operatorname{cotg}\varphi$; $\frac{a-b}{\sin\varphi}$; 39,6; 44,5.

33. $a - b \,.\, \operatorname{tg}\alpha = 50$.

34. $\operatorname{tg}\varphi = \sqrt{\frac{3b-2a}{b+2a}}$, $\varphi = 5^0\ 41',6$; $x = a \,.\, \operatorname{tg}\varphi = 29{,}9$.

35. $17^0\ 45'$. **36.** a) $63^0\ 26'\ 5'',7$; b) $26^0\ 33'\ 54'',3$.

37. $\operatorname{tg}\alpha = \frac{n}{n-1}$; $48^0\ 0'$.

38. $a \,.\, \operatorname{cotg}\alpha$; a) 27,598; b) 6,173; c) 5,1395.

39. 5,5469; 40,1580. **40.** $h : l = \operatorname{cotg}\varphi$; $51^0\ 41'\ 2'',3$.

41. a) $\frac{a \,.\, \sin(\alpha - \beta)}{\sin\alpha \,.\, \sin\beta}$; 14,1. b) 0,3. **42.** $0^0\ 1'\ 8'',76$.

43. 98,218 mal. **44.** $0^{cm},004848$. **45.** $7^0\ 12'\ 16'',6$.

46. 46410^m. **47.** 3829,5 Meilen. **48.** 9,3001 Meilen.

49. 66^0. **50.** 285. **51.** 1904 ,3. **52.** $4610^m,7$.

53. $\sqrt{a(2r \,.\, 7420{,}16 + a)}$ Meter. **54.** 53,201 □Ml.

55. $2^0\ 12'$; 4281. **56.** a) 8748^m; b) 36120^m.

57. $99^m,234$. **58.** 7791^m. **59.** 77,74 M.

60. 192308 M. **61.** $0^0\ 16'\ 0'',48$. **62.** 111,86 mal.

63. 51808,7 M.; 234,21 M. **64.** 399,225 mal.

65. $23^0\ 51'\ 55''$. **66.** $23^0\ 50'$. **67.** $6^0\ 22'\ 45'',6$.

68. $\frac{r \,.\, \cos\varphi \,.\, (b-a)\pi}{180^0} = 20$ Ml.

69. $\frac{a \cos\alpha \cos\beta}{\sin(\beta-\alpha)}$; 20^m. **70.** $\frac{h \sin(\alpha-\beta)}{\sin\alpha \sin\beta} = 12$.

71. $\operatorname{tg}\alpha = h : d$, $x = d \operatorname{tg}(\alpha + z) - h = 1^m,358$.

72. $\sqrt{d\,(a \operatorname{cotg}\beta - d) + \frac{1}{4}a^2} - \frac{1}{2}a = 750^m$.

73. $h = \frac{1}{2}a\sqrt{2} \,.\, \operatorname{tg}\alpha$; $x = \alpha$; 1700^m.

74. $h = a \operatorname{tg}\alpha : (\operatorname{tg}\beta - \operatorname{tg}\alpha)$; $d = h \cot\alpha$; $h = 500$, $d = 2000$.

75. $h \sin(\alpha - \beta) : \cos\alpha \sin\beta$; 3.

76. $\operatorname{tg}\varphi = \frac{a}{b} \sin 67^0\ 30'$; 29^0.

77. $2a - b + d \operatorname{tg}\alpha \,.\, \cos\varphi = 35$.

78. 0,4. **79.** $\sin\alpha = \frac{1}{n}$. **80.** $\sin\alpha = b^2\sqrt{2} : 2a^2$.

81. $\frac{1}{2}\sqrt{2} \,.\, h \cot\alpha$. **82.** $\sin x = b \operatorname{tg}\alpha : h$.

83. $a \operatorname{tg}\alpha : \sqrt{2}$; 1,043. **84.** $4^0\ 34'\ 52''$.

85. $10000 : 2882 = 3,4698 : 1$; $73^0\ 15'$.

86. $a \sin\alpha$; $a \cos\alpha$; 3; 4.

87. $R = 100$, $\alpha = 223^0$.

88. $p \sin\alpha = 13,6$; $p \cos\alpha = 27,3$.

89. $P = \sqrt{p^2 + q^2} = 139,02$; $\operatorname{tg}\alpha = p : q$; $\alpha = 44^0$.

90. 2,3334 Kgr. **91.** 22^0.

92. $\frac{1}{2}gt^2 \sin\alpha = 10^m$. **93.** 38^0.

94. $x = ap \sin\alpha : q \sin\beta$; $D = \sqrt{p^2 + q^2 + 2pq \cos(\alpha + \beta)}$.

95. 0,174312. **96.** $0^0\ 42'\ 58'',4$. **97.** $73^0\ 18'\ 2'',6$.

98. $17^0\ 15'$. **99.** $a^m \,.\, \cos\alpha$.

100. $x = \frac{\alpha\pi}{180^0} \,.\, \frac{r}{a}$; $\sin\frac{1}{2}\alpha = \frac{b}{2r}$. **101.** 6.

102. $5^0\ 8'\ 33'',8$ für $p = 9$. **103.** 0,083624; $3^0\ 10'\ 47'',4$.

104. $1^m,1048$. **105.** $c \,.\, \sin\alpha \,.\, \sin\beta : \sin(\beta - \alpha)$.

106. 2650,4. **107.** $a : \cos\alpha$.

§. 16.

1. $h = \frac{1}{2}c \,.\, \sin 2\alpha = 54{,}6$; $q = c \,.\, \cos\alpha^2 = 82{,}81$; $p = c \,.\, \sin\alpha^2 = 36$.

2. a) $\frac{1}{2}a^2 \operatorname{cotg}\alpha$; b) $\frac{1}{2}a^2 \operatorname{tg}\beta$; c) $\frac{1}{4}c^2 \sin 2\alpha$.

3. a) $a\sqrt{2} \,.\, (45^0 - \frac{1}{2}\alpha) : 2\cos\frac{1}{2}\alpha = 33$;
b) $c\sqrt{2} \,.\, \sin\frac{1}{2}\alpha \,.\, \sin(45^0 - \frac{1}{2}\alpha) = 13$.

4. $a : (1 + \operatorname{tg}\frac{1}{2}\beta) = 34$.

5. a) $\frac{1}{2}a\sqrt{1 + 4\operatorname{cotg}\alpha^2} = 241$;
b) $\frac{1}{2}b\sqrt{4 + \operatorname{tg}\alpha^2} = 289$;
c) $\frac{1}{2}c\sqrt{1 + 3 \,.\, \cos\alpha^2} = 421$.

6. $a \,.\, \cot\alpha : \cos\frac{1}{2}\alpha = b : \cos\frac{1}{2}\alpha = c\cos\alpha : \cos\frac{1}{2}\alpha = 257$.

7. 109; $56^0\ 36'\ 4'',4$; $21^0\ 58'\ 39'',3$.

8. $\sin\dfrac{n\alpha}{m+n}\cos\alpha : \sin\dfrac{m\alpha}{m+n} = 65$;
$\sin\dfrac{m\alpha}{m+n} : \sin\dfrac{n\alpha}{m+n}\cos\alpha = 43{,}113 : 1$.

9. $\dfrac{m}{m+n}c \,.\, \operatorname{cotg}\alpha$; $m : n = \sin\alpha : (\sqrt{2} - \sin\alpha)$.

10. $2\sin 54^0 : 1 = 1{,}61804 : 1$.

§. 17.

1. $\operatorname{tg}\beta = h : p = \operatorname{cotg}\alpha$; $\alpha = 3^0\ 3'\ 22'',0$;
$\beta = 86^0 56' 38'',0$; $a = \sqrt{p^2 + h^2} = p : \cos\beta = 89{,}608$;
$c = (p^2 + h^2) : p = p : \cos\beta^2 = 1680{,}77$;
$b = h\sqrt{p^2 + h^2} : p = p\sin\beta : \cos\beta^2 = 1678{,}38$;
$F = \frac{1}{2}ab = h(p^2 + h^2) : 2p = 75198{,}3$.

2. $\sin\beta = h : a$, $\beta = 55^0\ 17'\ 31''$; $\alpha = 34^0\ 42'\ 29''$;
$b = ah : \sqrt{a^2 - h^2} = a\operatorname{tg}\beta = 231$;
$c = a^2 : \sqrt{a^2 - h^2} = a : \cos\beta = 281$;
$F = \frac{1}{2}ab = \frac{1}{2}a^2h : \sqrt{a^2 - h^2} = 18480$.

3. $\cos\beta = p : a$, $\beta = 36^0\ 52'\ 11'',6$, $\alpha = 53^0\ 7'\ 48'',4$;
$c = a^2 : p = a : \cos\beta = 2{,}5$;
$b = a\sqrt{a^2 - p^2} : p = a\operatorname{tg}\beta = 2$;
$F = \frac{1}{2}ab = \frac{1}{2}a^2\sqrt{a^2 - p^2} : p = 1{,}5$.

4. $\beta = 22^0\ 37'\ 11'',5$; $b = h : \sin\alpha = 65$;
$a = b \operatorname{tg}\alpha = h : \cos\alpha = 156$;
$c = b : \cos\alpha = 2h : \sin 2\alpha = 169$;
$F = \frac{1}{2} c h = h^2 : \sin 2\alpha = 5070$.

5. $\beta = 61^0\ 55'\ 39'',1$; $a = p : \sin\alpha = 136$;
$b = p \cos\alpha : \sin\alpha^2 = 255$; $c = p : \sin\alpha^2 = 289$;
$F = \frac{1}{2} p^2 \cos\alpha : \sin\alpha^3 = 17340$.

6. $\operatorname{tang}\alpha = m : n$, $\alpha = 47^0\ 15'\ 31'',5$;
$a = cm : \sqrt{m^2 + n^2} = 2{,}016$; $b = cn : \sqrt{m^2 + n^2} = 1{,}863$;
$F = \frac{1}{2} c^2 mn : (m^2 + n^2) = 1{,}877904$.

7. $\operatorname{tang}\alpha = m : n$, $\alpha = 73^0\ 44'\ 23'',3$;
$a = h \sqrt{m^2 + n^2} : n = 600$;
$b = h \sqrt{m^2 + n^2} : m = 175$; $c = h\,(m^2 + n^2) : mn = 625$;
$F = \frac{1}{2} h^2 (m^2 + n^2) : mn = 52500$.

8. $\alpha = 45^0 + \frac{1}{2}\delta = 87^0\,3'\,44'',6$; $\beta = 45^0 - \frac{1}{2}\delta = 2^0\,56'\,15'',4$;
$a = c \sin\alpha = 22{,}80$; $b = c \cos\alpha = 1{,}17$;
$F = \frac{1}{4} c^2 \cos\delta = 13{,}338$.

9. $b = 2F : a = 2{,}07$; $c = \sqrt{a^2 + 4F^2 : a^2} = 3{,}05$;
$\operatorname{tang}\alpha = a^2 : 2F$, $\alpha = 47^0\,15'\,31'',5$, $\beta = 42^0\,44'\,28'',5$.

10. $a = \sqrt{p \,.\, c} = 100$; $b = \sqrt{c\,(c - p)} = 621$;
$\operatorname{tg}\alpha = \sqrt{p : (c - p)}$, $\alpha = 9^0\ 8'\ 52'',3$,
$\beta = 80^0\ 51'\ 7'',7$; $F = \frac{1}{2} c \sqrt{p\,(c - p)} = 31050$.

11. $a = \frac{1}{2}\sqrt{c\,(c + 2h)} + \frac{1}{2}\sqrt{c\,(c - 2h)} = 187$;
$b = \frac{1}{2}\sqrt{c\,(c + 2h)} - \frac{1}{2}\sqrt{c\,(c - 2h)} = 84$;
$\sin 2\alpha = 2h : c$, $\operatorname{tg}\frac{1}{2}(\alpha - \beta) = \sqrt{(c - 2h) : (c + 2h)}$;
$\alpha = 65^0\,48'\ 37'',7$, $\beta = 24^0\ 11'\ 22'',3$;
$F = \frac{1}{2} c h = 7854$.

12. $a = \sqrt{\frac{1}{2} c\,(c + d)} = 35$; $b = \sqrt{\frac{1}{2} c\,(c - d)} = 12$;
$\operatorname{tg}\alpha = \sqrt{(c + d) : (c - d)}$, $\alpha = 71^0\ 4'\ 31'',3$,
$\beta = 18^0\ 55'\ 28'',7$; $F = \frac{1}{4} c \sqrt{c^2 - d^2} = 210$.

13. $c = 2\sqrt{\frac{1}{2} a^2 + \frac{1}{16} d^2} - \frac{1}{2} d = 481$; $b = \sqrt{c^2 - a^2} = 319$;
$\sin\alpha = a : c$, $\alpha = 48^0\ 27'\ 19'',7$, $\beta = 41^0\ 32'\ 40'',3$;
$F = \frac{1}{2} ab = 57420$.

14. $c = \sqrt{d^2 + 4h^2} = 210{,}232$; $\operatorname{tang} 2\alpha = -\frac{2h}{d}$.
$a = \sqrt{\frac{1}{2}c(c+d)} = 207{,}348$;
$b = \sqrt{\frac{1}{2}c(c-d)} = 34{,}773$;
$\alpha = 80^0\ 28'\ 47'',7$; $F = 3605$.

15. $b = \sqrt{\frac{2nF}{m}} = 17{,}5$; $a = \sqrt{\frac{2mF}{n}} = 28{,}8$;
$c = \sqrt{\frac{2(m^2+n^2)F}{mn}} = 33{,}7$, $\operatorname{tg} \alpha = \frac{m}{n}$,
$\alpha = 58^0\ 42'\ 55'',8$, $\beta = 31^0\ 17'\ 4'',2$.

16. $a = \frac{1}{2}s + \sqrt{\frac{1}{4}s^2 - 2F} = 352$;
$b = \frac{1}{2}s - \sqrt{\frac{1}{4}s^2 - 2F} = 135$;
$c = \sqrt{s^2 - 4F} = 377$, $\sin 2\alpha = 4F : (s^2 - 4F)$,
$\alpha = 69^0\ 1'\ 1'',4$, $\beta = 20^0\ 58'\ 58'',6$.

17. $a = \sqrt{ps + \frac{1}{4}p^2} - \frac{1}{2}p = 28{,}545$;
$c = s + \frac{1}{2}p - \sqrt{ps + \frac{1}{4}p^2} = 29{,}929$;
$b = 8{,}996$; $\alpha = 72^0\ 30'\ 27'',6$, $\beta = 17^0\ 29'\ 32'',4$;
$F = 128{,}395$ (41).

18. $c = \frac{s^2}{2s-q} = 342{,}25$; $a = \frac{s(s-q)}{2s-q} = 192{,}40$;
$b = s\sqrt{\frac{q}{2s-q}} = 283{,}05$; $F = 27229{,}4$.

19. $a = \frac{1}{2}s + \frac{1}{2}\sqrt{2c^2 - s^2} = 416$;
$b = \frac{1}{2}s - \frac{1}{2}\sqrt{2c^2 - s^2} = 87$, $\cos(\alpha - 45^0) = s\sqrt{2} : 2c$,
$\alpha = 78^0\ 11'\ 15'',8$, $\beta = 11^0\ 48'\ 44'',2$. $F = 18096$.

20. $c = \frac{s^2+b^2}{2s} = 5{,}86$; $a = \frac{s^2-b^2}{2s} = 1{,}36$; $\sin \alpha = \frac{s^2-b^2}{s^2+b^2}$,
$\alpha = 13^0\ 25'\ 10'',8$, $\beta = 76^0\ 34'\ 49'',2$; $F = 3{,}876$.

21. $c = s\sqrt{2} : 2\cos(\alpha - 45^0) = 305$; $a = 136$; $b = 273$;
$\beta = 63^0\ 31'\ 8'',3$; $F = 18564$.

22. $c = s : 2\cos\frac{1}{2}\beta^2 = 6{,}50$; $a = 4{,}08$; $b = 5{,}06$;
$F = 10{,}3224$.

23. $\sin \alpha = \frac{n}{m}$, $\alpha = 50^0\ 24'\ 8'',1$, $\beta = 39^0\ 35'\ 51'',9$;
$a = \frac{ns}{n + \sqrt{m^2 - n^2}} = 81{,}6$; $c = \frac{ms}{n + \sqrt{m^2 - n^2}} = 105{,}9$;
$b = 67{,}5$; $F = 2754$.

24. $\cos(\alpha - 45^0) = s\sqrt{2} : 2a$; $\alpha = \beta = 45^0$,
$b = a = 6\sqrt{2}$, $c = 12$, $F = 36$.

25. $a = s\sqrt{2} : 2\cos(\alpha - 45^0) = 2$; $b = 3$;
$c = 3{,}6055$; $F = 3$.

26. $a = \frac{s^2 + p^2}{2s} = 3$; $\sin\alpha = \frac{2ps}{s^2 + p^2}$, $\operatorname{tg}\frac{1}{2}\alpha = \frac{p}{s}$,
$\alpha = 35^0\ 15'\ 52'',9$;
$b = 3\sqrt{2} = 4{,}24264$; $c = 3\sqrt{3} = 5{,}19615$;
$F = 4{,}5\sqrt{2} = 6{,}36396$.

27. $a = s : 2\cos\frac{1}{2}\alpha^2 = 5$; $b = 9$; $c = 10{,}2957$; $F = 22{,}5$.

28. $a = u\sin\frac{1}{2}\alpha : \sqrt{2}\,.\cos(45^0 - \frac{1}{2}\alpha) = 3{,}4$;
$b = 1{,}89$; $c = 3{,}89$; $F = 3{,}2130$.

29. $a = d\cos\frac{1}{2}\alpha : \sqrt{2}\,.\sin(45^0 - \frac{1}{2}\alpha) = 40{,}8$;
$b = 14{,}5$; $c = 43{,}3$; $F = 295{,}8$.

30. $c = -d : \cos 2\alpha = 17{,}6917$; $a = 13$; $b = 12$; $F = 78$.

31. $c = 2\sqrt{F : \sin 2\alpha} = 48{,}5$; $a = \sqrt{2F : \operatorname{cotg}\alpha} = 47{,}6$;
$b = \sqrt{2F : \operatorname{tg}\alpha} = 9{,}3$.

32. $\cos\alpha = (\sqrt{p^2 + 4b^2} - p) : 2b$, $\alpha = 19^0$;
$c = \frac{1}{2}(\sqrt{p^2 + 4b^2} + p) = 24{,}7344$;
$a = 8{,}0527$; $F = 94{,}1625$.

33. $b = \varrho(1 + \operatorname{cotg}\frac{1}{2}\alpha) = \varrho\sqrt{2}\,.\cos(45^0 - \frac{1}{2}\alpha) : \sin\frac{1}{2}\alpha = 323$;
$a = \varrho\sqrt{2}\,.\cos\frac{1}{2}\alpha : \sin(45^0 - \frac{1}{2}\alpha) = 36$;
$c = \varrho\sqrt{2} : 2\sin\frac{1}{2}\alpha\sin(45^0 - \frac{1}{2}\alpha) = 325$;
$F = \varrho^2\cot\frac{1}{2}\alpha\operatorname{tg}(45^0 + \frac{1}{2}\alpha) = 5814$.

34. $\cos(\alpha - 45^0) = \frac{1}{2}\sqrt{2}\,.(h - \varrho) : \varrho$, $\alpha = 58^0\ 27'\ 6'',4$;
$c = 2\varrho^2 : (h - 2\varrho) = 349$; $a = 299$, $b = 180$,
$F = 26910$.

35. $c = \frac{F - \varrho^2}{\varrho} = 37{,}3$; $a = \frac{1}{2}\,.\frac{F + \varrho^2}{\varrho} + \frac{1}{2}\sqrt{c^2 - 4F} = 27{,}5$;
$b = \frac{1}{2}\,.\frac{F + \varrho^2}{\varrho} - \frac{1}{2}\sqrt{c^2 - 4F} = 25{,}2$;
$\cos(\alpha - 45^0) = (F + \varrho^2)\sqrt{2} : 2(F - \varrho^2)$; $\alpha = 47^0\ 29'\ 56'',4$.

36. $\sin 2\alpha = \frac{2h}{s^2}(h + \sqrt{s^2 + h^2})$, $\alpha = 29^0$;
$a = h : \cos\alpha = 201{,}195$; $b = h : \sin\alpha = 362{,}967$;
$c = 2h : \sin 2\alpha = 415$; $F = 36514$.

37. $\sin 2\alpha = \frac{2h}{d^2}(\sqrt{d^2+h^2}-h)$, $\alpha = 2^0$;
$a = h : \cos\alpha = 1{,}5356$; $b = h : \sin\alpha = 43{,}9730$;
$c = 2h : \sin 2\alpha = 44$; $F = 33{,}7623$.

38. $\cot 2\alpha = -\frac{1}{4}d^2 : F$, $c^2 = 4F : \sin 2\alpha$;
$\alpha = 46^0\ 54',46$, $\beta = 43^0\ 5',54$,
$c = 25{,}507$, $a = 18{,}627$, $b = 17{,}426$.

39. $\cos(\alpha - 45^0) = f : d\sqrt{2}$, $\alpha = 68^0\ 8'$;
$c = -f : \cos 2\alpha = 80{,}544$;
$a = 74{,}748$; $b = 29{,}998$; $F = 1121{,}16$.

40. $c = s - 2\varrho = 349{,}002$; $\sin(45^0+\alpha) = \frac{1}{2}s\sqrt{2} : (s - 2\varrho)$,
$a - b = \sqrt{s^2 - 8\varrho s + 8\varrho^2}$, $a = 339$, $b = 82{,}952$;
$\alpha = 76^0\ 15'$; $F = \varrho(s-\varrho) = 167{,}694$.

41. $\cot\frac{1}{2}\alpha = (2\varrho - d + \sqrt{8\varrho^2 + d^2}) : 2\varrho$;
$\alpha = 70^0\ 35'\ 46'',7$, $b = 10{,}6835$;
$a = 30{,}3258$, $c = 32{,}152$; $F = 161{,}99$.

42. $a = \sqrt[3]{m^2 \cdot p} = 476$; $c = m^2 : a = 485$; $b = 93$;
$\alpha = 78^0\ 56'\ 41'',7$; $F = 22134$.

43. $c = \frac{1}{4}(s + \sqrt{8a^2 + s^2}) = 2500$; $b = 700$;
$\alpha = 73^0\ 44'\ 23'',3$; $F = 840000$.

44. $\cos\beta = (d + \sqrt{8s^2 + 8sd + d^2}) : 4s$, $\beta = 77^0\ 58'\ 55''$;
$c = (4s + d \pm \sqrt{8s^2 + 8sd + d^2}) : 2 = 365$;
$a = 76$; $b = 357$; $F = 13566$.

45. $\cot\alpha = \sqrt{\frac{s}{p} - \frac{3}{4}} - \frac{1}{2}$, $\alpha = 77^0\ 4'\ 23'',7$;
$a = \frac{p}{\sin\alpha} = 444$, $c = 455{,}54$, $b = 101{,}91$; $F = 22623$.

46. $c = 2s : (2 + \sin 2\alpha) = 376{,}93$; $a = 151{,}97$;
$b = 344{,}94$; $F = 26211$.

47. $c = (m^2 - 2\varrho^2) : 2\varrho = 397$; $\sin 2\alpha = 2m^2 : c^2$,
$\alpha = 35^0\ 3'\ 4'',1$; $a = 228$; $b = 325$; $F = 37050$.

48. $a = s - \sigma + \sqrt{2\sigma(\sigma - s)} = 304$; $b = s - a = 297$;
$c = 425$; $\alpha = 45^0\ 40'\ 2'',3$; $F = 45144$.

49. $a = s : 2\sin\frac{3}{2}\alpha \cdot \cos\frac{1}{2}\alpha$; $b = 2a\cos\alpha$;
a) $a = 125$, $b = 88$, $F = 5148$;
b) $a = 2{,}5277$, $b = 1{,}7291$, $F = 2{,}0535$;
c) $a = 19{,}946$, $b = 15{,}957$, $F = 145{,}847$.

50. $\cos\alpha = 0{,}6$; $\alpha = 53^0\ 7'\ 48'',4$; $\beta = 73^0\ 44'\ 23'',2$.

51. $b^3 - \frac{F}{\varrho} b^2 + 4F\varrho = 0$; $b^3 - 8b^2 = -72$; $b = 6$; $a = 5$; $\alpha = 53^0\ 7'\ 48'',4$; $\beta = 73^0\ 44'\ 23'',2$.

52. $b^3 - \frac{u^2 - h^2}{2u} \cdot b^2 - h^2 \cdot b + \frac{uh^2}{2} = 0$;
$169b^3 - 2842b^2 - 14400b + 259200 = 0$;
$b = 10$; $a = 13$; $\alpha = 67^0\ 22'\ 48'',5$; $\beta = 45^0\ 14'\ 23'',0$;
$F = 60$ oder $b = 16{,}2531$, $a = 9{,}8735$, $\alpha = 34^0\ 36',4$;
$F = 45{,}5700$.

§. 19.

1. $a : \sin\alpha = b : \sin\beta = c : \sin\gamma$.

3. $\sin\frac{m \cdot 180^0}{m+n+p} : \sin\frac{n \cdot 180^0}{m+n+p} : \sin\frac{p \cdot 180^0}{m+n+p}$;

a) $1 : \sqrt{3} : 2 = 1 : 1{,}73206 : 2$.

b) $2\sqrt{2} : 2\sqrt{3} : 2 + \sqrt{6} = 0{,}70711 : 0{,}86603 : 0{,}96593$.

c) $0{,}74314 : 0{,}86603 : 0{,}95106$.

4. $\cos\alpha = \frac{n^2 + p^2 - m^2}{2np}$, $\cos\beta = \frac{m^2 + p^2 - n^2}{2mp}$,
$\cos\gamma = \frac{m^2 + n^2 - p^2}{2mn}$.

a) c. $28^0\ 57'\ 17''$; $46^0\ 34'\ 4''$; $104^0\ 28'\ 39''$.

b) c. $26^0\ 23'$; $36^0\ 20'$; $117^0\ 17'$.

c) c. $14^0\ 38'$; $34^0\ 37'$; $130^0\ 45'$.

5. $\cos\beta = \frac{1}{2}a : b$; $\beta = 31^0\ 30'\ 0''$; $\alpha = 63^0\ 0'\ 0''$.

9. $a^2 + b^2 - ab$; $a^2 + b^2 - ab\sqrt{2}$.

10. $c^2 + c_1^2 = 2(a^2 + b^2)$.

19. Den Sinussatz. **20.** Die Formel für $\operatorname{tg}\frac{1}{2}\alpha$ durch die 3 Seiten. **21.** Der allg. pyth. Lehrsatz. **22.** Sie ist gleich dem Radius des einbeschriebenen Kreises. **23.** Den Sinussatz.

25. $\frac{\sin\frac{1}{2}\alpha}{\sin\frac{1}{2}\beta} = \sqrt{\frac{a(a-b+c)}{b(b+c-a)}}$, $\frac{\cos\frac{1}{2}\alpha}{\cos\frac{1}{2}\beta} = \sqrt{\frac{a(b+c-a)}{b(a-b+c)}}$, u. s. w.

26. $\operatorname{tang}\alpha = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{s(s-a) - (s-b)(s-c)}$.

§. 20.

24. $\alpha \sin \gamma : \sin (\beta + \gamma) = 300.$

25. 3989,5 und 4555,9 Schritte.

26. $a \sin \alpha : \sin \beta = 34^m.$ **27.** 2; 3. **28.** $d = 10,\ a = 5.$

29. $a = d \sin \psi : \sin (\psi + \varphi),\ c = d \sin \varphi : \sin (\psi + \varphi),$
$b = a - 2c \cos (\varphi + \psi).$

30. $(a - b) \sin \beta : \sin (\alpha + \beta);\ (a - b) \sin \alpha : \sin (\alpha + \beta).$

31. $d = a \sin \varphi : \sin (\beta + \varphi);$
$c = a \sin \beta \sin \varphi : \sin (\beta + \varphi) \sin \alpha;$
$b = a \sin \beta \sin (\alpha - \varphi) : \sin (\beta + \varphi) \sin \alpha.$

32. $b = a \sin \beta : \sin (\alpha + \beta);\ c = a \sin (\alpha - \gamma) : \sin (\alpha + \beta);$
$d = a \sin \gamma : \sin (\alpha + \beta);\ c = a \sin \alpha : \sin (\alpha + \beta);$
$f = a \sin (\alpha + \beta - \gamma) : \sin (\alpha + \beta).$

33. $e \sin \beta : \sin (\alpha + \beta),\ e \sin a : \sin (\alpha + \beta),$
$e \sin \delta : \sin (\gamma + \delta),\ e \sin \gamma : \sin (\gamma + \delta).$

34. $\sin 26^0\, 40' : \sin 33^0\, 20' = 1{,}22439 : 1 = 1 : 0{,}81663.$

35. $x = \frac{a \sin \beta}{\sin (\alpha + \beta)} \cdot \frac{\sin [60^0 - \frac{1}{3} (\alpha + \beta)]}{\sin [60^0 + \frac{1}{3}(2\alpha - \beta)]};$
$z = \frac{a \sin \alpha}{\sin (\alpha + \beta)} \cdot \frac{\sin [60^0 - \frac{1}{3} (\alpha + \beta)]}{\sin [60^0 + \frac{1}{3}(2\beta - \alpha)]}.$
$y = a - x - z;$ 2000; 791,93; 856,07.

36. $a \cos \frac{n - m}{n + m} \alpha : \cos \alpha = 10.$

37. $a \sin \gamma : 2 \cos \frac{1}{2} \beta \sin (\gamma + \frac{1}{2}\beta) = 16{,}86(04).$

38. $a \cos \frac{1}{2} (\beta - \gamma) : 2 \cos \frac{1}{2} \beta \cos \frac{1}{2} \gamma = 20{,}8333.$

39. $\sin \beta \sqrt{\frac{mac}{n \sin \varphi \sin (\beta + \varphi)}} = 13{,}611.$

40. $\sin \vartheta = \frac{a}{2r},\ \sin (2x + \vartheta) = \frac{b}{2r}.$

§. 21.

37. 380. **38.** 200^m.

39. 8 und 7,6158; 2,7954 und 17,7628.

40. $\frac{1}{2} \sqrt{d^2 + e^2 \mp 2de \cos \varphi};$ 32; 255; 90^0.

41. $a \sqrt{\frac{\cos \frac{1}{2} (\beta - \gamma)^2}{\cos \frac{1}{2} (\beta + \gamma)^2} - \frac{\sin \beta \sin \gamma}{\sin \frac{1}{2} (\beta + \gamma)^2}}.$

42. 14; 4; $67^0\, 22'\, 48'',5$.

43. $\sqrt{c^2 + (a - b)^2 - 2c(a - b) \cos \alpha}$.

44. $d = \dfrac{a \sin \beta + b \sin (\beta + \gamma)}{\sin \gamma}$,

$c = \dfrac{a \sin (\beta - \gamma) + b \sin \beta}{\sin \gamma}$, u. s. w.

45. $BC = 5$, $DA = 5{,}3851$, $A = 116^0\ 46'\ 14'',9$;
$B = 86^0\ 49'\ 15'',8$; $C = 88^0\ 12'\ 34'',2$; $D = 68^0\ 11'\ 55'',1$.

46. $19^0\ 6'\ 23'',5$; $21^0\ 47'\ 13''$;
$\sqrt{7} : 3 = 2{,}64575 : 3$.

47. a) $r = \dfrac{b + \frac{1}{2}a - \sqrt{b(a+b)} \cos \alpha}{\sin \alpha} = 4{,}038$;
$x^2 = t^2 + (a + b)^2 - 2(a + b)\, t \cos \alpha$, $x = 7{,}931$.
b) $r = b\sqrt{3}$, $F = 3b^2\pi$.
c) $\dfrac{a + b - 2\sqrt{ab} \,.\, \cos \alpha}{2 \sin \alpha^2} = 756$.

48. Die getheilte Seite sei a;
$x = \frac{1}{4}(a + 2b \cos \gamma \pm \sqrt{[a + 2b \cos \gamma]^2 - 8b^2})$, $y = a - x$.

49. $c^2 = R^2 + r^2 \pm 2Rr \cos \varphi$; $\frac{7}{5}R$ oder $\frac{\sqrt{19}}{5} R$.

§. 22.

26. $b = 92{,}325$, $e = 124{,}617$.

27. $a = 199{,}677$, $c = 235{,}022$.

28. $(a - b) \cos \alpha \pm \sqrt{c^2 - (a - b)^2 \sin \alpha^2}$.

30. $AB = 3{,}7543$, $CD = 7{,}8105$, $DA = 8{,}0726$,
$A = 110^0\ 0'\ 43'',8$; $B = 100^0\ 40'\ 23'',1$.

§. 23.

36. $62^0\ 4'\ 28''$, $59^0\ 52'\ 14''$, $58^0\ 3'\ 18''$.

37. 70^0, 50^0, 60^0. **38.** 60^0.

39. $\frac{1}{3}\sqrt{6c^2 + 3b^2 - 2a^2} = 16{,}4114$.

40. $b = \sqrt{\frac{1}{2}(d^2 + e^2) - a^2}$, $\cos \alpha = (d^2 - e^2) : 4ab$.

41. $\cos \alpha = \dfrac{a^2 + c^2 - d^2}{2ac}$, $b = \dfrac{d^2 - c^2}{a}$.

42. $\cos\alpha = \frac{(a-b)^2+d^2-c^2}{2(a-b)d}$, $\cos\beta = \frac{(a-b)^2+c^2-d^2}{2(a-b)c}$,
$e = \sqrt{\frac{a^2b-ab^2+ac^2+bd^2}{a-b}}$, $f = \sqrt{\frac{a^2b-ab^2+ad^2-bc^2}{a-b}}$.

43. $c = \sqrt{\frac{bf^2-a^2b-ab^2+ae^2}{a+b}}$, $d = \sqrt{\frac{be^2-a^2b-ab^2+af^2}{a+b}}$,
$\cos\alpha = \frac{a^2-b^2-e^2+f^2}{2(a+b)d}$, $\cos\beta = \frac{a^2-b^2+e^2-f^2}{2(a+b)c}$.

45. $\alpha = 88^\circ\ 51'\ 10''$, $\beta = 89^\circ\ 12'\ 14''$, $\gamma = 70^\circ\ 20'\ 46''$, $\delta = 111^\circ\ 35'\ 50''$.

§. 24.

32. 1050. **33.** 276. **34.** 2712. **35.** $ab \sin\alpha$.

36. $\frac{1}{4}(a^2 - b^2)\operatorname{tang}\alpha$. **37.** $\frac{1}{2}(a+b)c \,.\, \sin\alpha$.

38. 1740.

43. $\frac{ab+cd}{ab-cd}\sqrt{s(s-c-d)(s-b-d)(s-a-d)}$,
$2s = a+b+c+d$.

45. Die gegebenen Seiten schliessen im grössten Dreieck einen rechten Winkel ein.

46. $a\cos\beta + b\cos\alpha = c$. **47.** $\frac{1}{2}a^2 \sin\beta \sin\gamma : \sin\alpha$.

48. $AX = 8{,}636$, $AY = 6{,}092$.

49. $2 \sin\alpha \sin\beta \sin\gamma : \pi$.

§. 26.

1. a) $a = 2r\sin\alpha = 44$, $b = 2r\sin\beta = 39$,
$c = 2r\sin\gamma = 17$.

b) $h_a = 2r\sin\beta\sin\gamma = 15$; $h_b = 2r\sin\alpha\sin\gamma = 16{,}9231$;
$h_c = 2r\sin\alpha\sin\beta = 38{,}8235$;
$F = 2r^2\sin\alpha\sin\beta\sin\gamma = 330$.

c) $h_a' = 2r\cos\alpha = -4{,}2$; $h_b' = 2r\cos\beta = 20{,}8$;
$h_c' = 2r\cos\gamma = 40{,}8$.

d) $h_a'' = 2r\cos\beta\cos\gamma = 19{,}2$; $h_b'' = 2r\cos\alpha\cos\gamma = -3{,}8769$; $h_c'' = 2r\cos\alpha\cos\beta = -1{,}9765$.

e) $q_a = 2r\cos\beta\sin\gamma = 8$; $q_b = 2r\sin\alpha\cos\gamma = 40{,}6154$;
$q_c = 2r\cos\alpha\sin\beta = -3{,}7058$; $p_a = 2r\sin\beta\cos\gamma = 36$;
$p_b = 2r\cos\alpha\sin\gamma = -1{,}6154$; $p_c = 2r\sin\alpha\cos\beta = 20{,}7058$.

f) $\alpha_1 = 180^0 - 2\alpha$ oder $2\alpha - 180^0 = 10^0\ 54'\ 18'',8$; $\beta_1 = 180^0 - 2\beta$ od. $2\beta = 123^0\,51'\,18'',2$; $\gamma_1 = 180^0 - 2\gamma$ oder $2\gamma = 45^0\ 14'\ 23'',0$.

g) $a_1 = r \sin 2\alpha = 4,181$; $b_1 = r \sin 2\beta = 18,353$; $c_1 = r \sin 2\gamma = 15,693$.

h) $F_1 = \frac{1}{2} r^2 \sin 2\alpha \sin 2\beta \sin 2\gamma = 27,243$.

i) $r_1 = \frac{1}{2} r = 11,05$.

2. a) $\varrho = 4r \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta \sin \frac{1}{2}\gamma = 9\frac{6}{7} = 9,85714$.

b) $\varrho_a = 4r \sin \frac{1}{2}\alpha \cos \frac{1}{2}\beta \cos \frac{1}{2}\gamma = 161$;
$\varrho_b = 4r \cos \frac{1}{2}\alpha \sin \frac{1}{2}\beta \cos \frac{1}{2}\gamma = 42$;
$\varrho_c = 4r \cos \frac{1}{2}\alpha \cos \frac{1}{2}\beta \sin \frac{1}{2}\gamma = 14$.

c) $s = 4r \cos \frac{1}{2}\alpha \cos \frac{1}{2}\beta \cos \frac{1}{2}\gamma = 98$;
$s - a = 4r \cos \frac{1}{2}\alpha \sin \frac{1}{2}\beta \sin \frac{1}{2}\gamma = 6$;
$s - b = 4r \sin \frac{1}{2}\alpha \cos \frac{1}{2}\beta \sin \frac{1}{2}\gamma = 23$;
$s - c = 4r \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta \cos \frac{1}{2}\gamma = 69$.

d) $OO_a = 4r \sin \frac{1}{2}\alpha = 176,94$; $OO_b = 4r \sin \frac{1}{2}\beta = 81,598$; $OO_c = 4r \sin \frac{1}{2}\gamma = 29,295$.

e) $O_a O_b = 4r \cos \frac{1}{2}\gamma = 107,70$; $O_a O_c = 4r \cos \frac{1}{2}\beta = 190,39$; $O_b O_c = 4r \cos \frac{1}{2}\alpha = 204,64$.

4. a) $v_a = 2r \sin \frac{1}{2}\alpha \sin \gamma : \cos \frac{1}{2}(\beta - \gamma) = 26,5$;
$u_a = 2r \sin \frac{1}{2}\alpha \sin \beta : \cos \frac{1}{2}(\beta - \gamma) = 25,5$.
$v_b = 2r \sin \alpha \sin \frac{1}{2}\beta : \cos \frac{1}{2}(\alpha - \gamma) = 25\frac{9}{35}$;
$u_b = 2r \sin \frac{1}{2}\beta \sin \gamma : \cos \frac{1}{2}(\alpha - \gamma) = 25\frac{26}{35}$;
$v_c = 2r \sin \beta \sin \frac{1}{2}\gamma : \cos \frac{1}{2}(\alpha - \beta) = 26\frac{25}{103}$;
$u_c = 2r \sin \alpha \sin \frac{1}{2}\gamma : \cos \frac{1}{2}(\alpha - \beta) = 26\frac{78}{103}$.

b) $w_a = 2r \sin \beta \sin \gamma : \cos \frac{1}{2}(\beta - \gamma) = 45,024$;
$w_b = 2r \sin \alpha \sin \gamma : \cos \frac{1}{2}(\alpha - \gamma) = 45,888$;
$w_c = 2r \sin \alpha \sin \beta : \cos \frac{1}{2}(\alpha - \beta) = 44,157$.

c) $w_a' = 4r \sin \frac{1}{2}\beta \sin \frac{1}{2}\gamma = 30,016$;
$w_b' = 4r \sin \frac{1}{2}\alpha \sin \frac{1}{2}\gamma = 30,886$;
$w_c' = 4r \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta = 29,154$.

d) $w_a'' = 4r \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta \sin \frac{1}{2}\gamma : \cos \frac{1}{2}(\beta - \gamma) = 15,008$;
$w_b'' = 4r \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta \sin \frac{1}{2}\gamma : \cos \frac{1}{2}(\alpha - \gamma) = 15,002$;
$w_c'' = 4r \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta \sin \frac{1}{2}\gamma : \cos \frac{1}{2}(\alpha - \beta) = 15,003$.

e) $a_2^2 = v_c^2 + u_b^2 - 2 v_c u_b \cos \alpha$, u. s. w. $a_2 = 25,967$; $b_2 = 25,864$; $c_2 = 26,114$;
$\alpha_2 = 59^0 46'\ 15''$; $\beta_2 = 59^0\ 44'\ 21''$; $\gamma_2 = 60^0\ 29'\ 24''$.

f) $F_2 = 292,25$. g) $r_2 = 15,003$.

5. $m_a = r\sqrt{2 \sin \beta^2 + 2 \sin \gamma^2 - \sin \alpha^2} = 94{,}43$;
$m_b = r\sqrt{2 \sin \alpha^2 + 2 \sin \gamma^2 - \sin \beta^2} = 206{,}93$;
$m_c = r\sqrt{2 \sin \alpha^2 + 2 \sin \beta^2 - \sin \gamma^2} = 292{,}66$;
$m_a' = \frac{2}{3} m_a = 62{,}953$; $m_b' = 137{,}953$; $m_c' = 195{,}107$;
$m_a'' = \frac{1}{3} m_a = 31{,}477$; $m_b'' = 68{,}977$; $m_c'' = 97{,}553$;
$\alpha_1 = 24^0\ 23'\ 38'',0$; $\alpha_2 = 120^0\ 49'\ 10'',1$;
$\beta_1 = 11^0\ 47'\ 3'',2$; $\beta_2 = 14^0\ 12'\ 18'',0$;
$\gamma_1 = 4^0\ 58'\ 38'',0$; $\gamma_2 = 3^0\ 49'\ 12'',7$;
$a' = 166$; $b' = 127{,}5$; $c' = 44{,}5$; $F' = 1618{,}5$;
$\alpha_3 = \alpha$; $\beta_3 = \beta$; $\gamma_3 = \gamma$.

6. Berechne $F = \sqrt{s(s-a)(s-b)(s-c)}$, $p_a = a(b^2+c^2-a^2) : 8F$, $p_b = b(a^2+c^2-b^2) : 8F$, $\varrho = F:s$, $\varrho_a = F:(s-a)$, $OM^2 = (\varrho - p_a)^2 + \frac{1}{4}(b-c)^2$, $O_a M^2 = (\varrho_a - p_b)^2 + \frac{1}{4}(a+c)^2$, u. s. w. 0,95763; 0,249825; 0,66224; 1,718578.

7. $\sqrt{b^2 + \frac{c^2}{(n+1)^2} - \frac{b^2+c^2-a^2}{n+1}} = 24$; $22^0\ 37'\ 11'',5$; $16^0\ 15'\ 36'',7$.

8. Derselbe Satz gilt für jeden dieser Kreise.

10. $a_1 = a : 2 \sin \frac{1}{2}\alpha$, $b_1 = b : 2 \sin \frac{1}{2}\beta$,
$c_1 = c : 2 \sin \frac{1}{2}\gamma$, $F_1 = F : 8 \sin \frac{1}{2}\alpha \,.\, \sin \frac{1}{2}\beta \,.\, \sin \frac{1}{2}\gamma$.

11. Die Dreiecke sind congruent..

13. $180^0 - 2\alpha$, $180^0 - 2\beta$, $180^0 - 2\gamma$.

14. Das Dreieck ist dem vorigen ähnlich, seine Seiten sind $a_1 = r(\operatorname{tg} \beta + \operatorname{tg} \gamma) = a : 2 \cos \beta \,.\, \cos \gamma$, u. s. w.

15. $\sin \gamma : \sin \beta : 2 \cos \frac{1}{2}(\beta - \gamma) \cos \frac{1}{2}\alpha : \sin \beta \operatorname{cotg} \frac{1}{2}\alpha \,.\, \operatorname{cotg} \frac{1}{2}(\beta - \gamma)$.

17. Ja, nur sind zwei Winkel des Dreiecks Differenzen statt Summen von mit den Transversalen gebildeten Winkeln.

§. 27.

1. $a = 1000$, $b = 879{,}38$, $c = 347{,}30$, $\beta = 60^0$, $\gamma = 20^0$.

2. $a = \frac{1}{3}(2s + 2d_1 + d_2) = 7{,}934936$;
$b = \frac{1}{3}(2s - d_1 + d_2) = 4{,}809527$;
$c = \frac{1}{3}(2s - d_1 - 2d_2) = 4{,}488927$;
$\alpha = 117^0\ 6'\ 58''$, $\beta = 32^0\ 38'\ 58''$, $\gamma = 30^0\ 14'\ 4''$.

3. $b = 370$, $c = 421$, $\alpha = 86^0\ 3'\ 0'',4$, $\beta = 43^0\ 1'\ 23'',5$.

4. $b = ns : (m + n) = 1$, $a = ms : (m + n) = 0{,}96886$, $c = 0{,}49389$, $\alpha = 72^0$, $\beta' = 79^0$.

5. $a + b = \sqrt{g^2 + 2f^2}$, $a - b = \sqrt{g^2 - 2f^2}$; $a = 120$, $b = 17$, $c = 113$, $\alpha = 110^0\ 26'\ 39''$, $\beta = 7^0\ 37'\ 41'',3$, $F = 900$.

6. $a + b = \sqrt{d^2 + 4f^2}$; $a = 240$, $b = 221$, $c = 29$, $\beta = 46^0\ 23'\ 49'',9$, $\gamma = 5^0\ 27'\ 9'',5$, $F = 2520$.

7. $a - b = \sqrt{2g^2 - s^2}$; $a = 87$, $b = 76$, $c = 65$, $\alpha = 75^0\ 44'\ 59'',9$, $\beta = 57^0\ 51'\ 10'',2$, $F = 2394$.

A.

1. $\sin\alpha = h : b$, $\sin\beta = h : a$, $c = \sqrt{a^2 - h^2} \pm \sqrt{b^2 - h^2}$.
   α) $c = 44{,}063\ (19{,}404)$, $\beta = 28^0\ 10'\ 44'',2$,
   $\alpha = 54^0 3' 0'',0 (125^0 57' 0'',0)$, $\gamma = 97^0 46' 15'',8 (25^0 52' 15'',8)$,
   $F = 374{,}536\ (164{,}934)$.
   β) $c = 21(9)$, $\beta = 28^0 4' 20'',0$, $\alpha = 53^0 7' 48'',2 (126^0 52' 11'',8)$,
   $\gamma = 98^0\ 47'\ 51'',8\ (25^0\ 3'\ 28'',2)$, $F = 84\ (36)$.

2. $\sin\alpha = h : b$, $a = h : \sin\beta$; $a = 317$, $c = 510$, $\alpha = 35\ 18'\ 0'',9$, $\gamma = 68^0\ 23'\ 7'',1$, $F = 78540$.

3. $\sin\alpha = h : b$, $\operatorname{tang}\beta = h : p$, $a = \sqrt{h^2 + p^2}$; $a = 1443$, $c = 1916\ (964)$, $\alpha = 11^0\ 3'\ 18'',3\ (168^0\ 56'\ 41'',7)$, $\beta = 3^0\ 41'\ 42'',8$, $\gamma = 165^0\ 14'\ 58'',9\ (7^0\ 21'\ 35'',5)$, $F = 89094\ (44826)$.

4. $\sin\alpha = h : b$; $a = 445$, $c = 606$, $\alpha = 44^0\ 29'\ 53'',0$, $\beta = 62^0\ 51'\ 32'',9$, $F = 119988$.

5. $\cos\alpha = q : b$; $\alpha = 62^0\ 51'\ 32'',9$, $\beta = 78^0\ 34'\ 43'',7$, $a = 404$, $c = 283$, $F = 56034$.

6. $b = \sqrt{q^2 + h^2} = 145$, $a = \sqrt{p^2 + h^2} = 617$, $c = p \pm q = 708\ (508)$, $\operatorname{tang}\alpha = \pm\ h : p$, $\operatorname{tang}\beta = h : q$; $\alpha = 46^0\ 23'\ 49'',9\ (133^0\ 36'\ 10'',1)$, $\beta = 9^0\ 47'\ 53'',5$, $\gamma = 123^0\ 48'\ 16'',6\ (36^0\ 35'\ 56'',4)$, $F = 38170\ (26670)$.

7. $b = h : \sin\alpha = 28{,}115$; $a = h : \sin\beta = 33{,}305$; $\gamma = 61^0\ 2'\ 40''$, $c = 31{,}512$, $F = 409{,}655$.

8. $c = p + q = 5$, $b = q : \cos\alpha = 0{,}8889$, $\operatorname{tang}\beta = q \operatorname{tang}\alpha : p$, $\beta = 9^0\ 16'\ 31'',5$, $\gamma = 114^0\ 57'\ 12'',5$, $a = 4{,}5596$, $F = 1{,}8373$.

9. $\gamma = \gamma_1 \pm \gamma_2 = 138^0\ 35'\ 21'',1\ (28^0 41' 29'',3)$, $b = h : \cos\gamma_2 = 1191$, $a = h : \cos\gamma_1 = 6175$, $\beta = 6^0\ 21'\ 34'',8$, $\alpha = 35^0\ 3'\ 4'',1\ (144^0\ 56'\ 55'',9)$, $c = 7112\ (5162)$, $F = 2432304\ (1765404)$.

10. $a = h : \sin\beta = 17{,}770$, $\operatorname{tang}\alpha = \pm h : (c - h \operatorname{cotg}\beta)$; $\alpha = 57^0\ 18'\ 43''$, $b = 19{,}011$, $\gamma = 58^0\ 28'\ 59''$, $F = 144$.

11. $\sin\alpha = h : b$; $\alpha = 32^0\ 31'\ 13'',5$, $\beta = 41^0\ 42'\ 32'',1$, $\gamma = 105^0\ 46'\ 14'',4$, $a^2 = b^2 + c^2 - 2c\sqrt{b^2 - h^2} = 505$, $F = 151872$.

12. $a = 2F : h_a = 11{,}25$, $b = 2F : h_b = 10$, $\sin\gamma = h_a h_b : 2F$, $\gamma_1 = 53^0\ 7'\ 48'',2$; $\alpha = 70^0\ 8'\ 41'',5$, $\beta = 56^0\ 43'\ 30'',2$, $c = 8{,}6350$.

13. $\sin\beta = h_a : c$, $\sin\alpha = h_b : c$; $a = 469{,}79\ (687{,}89)$, $b = 194{,}18\ (284{,}33)$, $\alpha = 65^0\ (115^0)$, $\beta = 22^0$, $\gamma = 93^0\ (43^0)$, $F = 45548\ (66694{,}3)$.

14. $a = h_b : \sin\gamma = 72{,}713$, $b = h_a : \sin\gamma = 107{,}293$, $c = 88{,}142$, $\alpha = 42^0\ 16'\ 52''$, $\beta = 83^0\ 4'\ 48''$, $F = 3181{,}22$.

15. $c = h_a : \sin\beta = 20$, $\sin\alpha = h_b \sin\beta : h_a$, $\alpha = 64^0\ 33'\ 10''\ (115^0\ 26'\ 50'')$, $\gamma = 64^0\ 33' 5'',3\ (13^0 39' 25'',3)$, $b = h_a : \sin\gamma = 17{,}1873\ (65{,}7329)$, $a = h_b : \sin\gamma = 20\ (38{,}2445)$, $F = 155{,}1\ (296{,}779)$.

16. $a = h : \sin\beta = 435$, $b = s - a = 149$, $\sin\alpha = h : b$, $\alpha = 20^0 0' 57'',5\ (159^0 59' 2'',5)$, $\gamma = 153^0 15' 4'',0\ (13^0 16' 59'',0)$, $c = 572\ (292)$, $F = 14586\ (7446)$.

17. $a = h : \sin\gamma = 267$, $b = s - a = 284$, $c = 125$, $\alpha_1 = 69^0\ 23'\ 25'',1$, $\gamma = 84^0\ 37'\ 13'',7$, $F = 16614$.

18. $b = h : \sin\alpha = 137$, $a = d + b = 1839$, $\beta = 3^0\ 16'\ 23'',4$, $\gamma = 125^0\ 41'\ 35'',0$, $c = 1924$, $F = 101010$.

19. $c = 2F : h = 716$, $b = s - c = 185$, $\sin\alpha = h : b$, $\alpha_1 = 17^0\ 56'\ 42'',9$, $a = 543$, $\beta = 6^0\ 1'\ 32'',1$, $\gamma = 156^0\ 1'\ 45'',0$, $F = 20406$.

20. $\operatorname{tang}\beta = h:(\frac{1}{2}c + \sqrt{m^2 - h^2})$, $\operatorname{tang}\alpha = h:(\frac{1}{2}c - \sqrt{m^2 - h^2})$, $\beta = 4^0\ 58'\ 44'',7$, $\alpha = 14^0\ 51'\ 46'',2$, $\gamma = 160^0\ 9'\ 29'',1$, $b = 269$, $a = 795$, $F = 36294$.

21. $\sin\alpha = h:b$; $\alpha_1 = 25^0\ 53'\ 14'',0$, $c = 2(\sqrt{m^2 - h^2} \pm \sqrt{b^2 - h^2}) = 968\ (392)$.

22. $\cos\varphi = h:w$, $b = h:\cos(\frac{1}{2}\gamma - \varphi)$, $a = h:\cos(\frac{1}{2}\gamma + \varphi)$, $\beta = 90^0 - \varphi - \frac{1}{2}\gamma$, $\alpha = 90^0 + \varphi - \frac{1}{2}\gamma$; $a = 795$, $b = 269$, $c = 1052$, $\alpha = 14^0\ 51'\ 46'',2$, $\beta = 4^0 58' 44'',7$, $F = 36294$.

23. $\sin\varphi = h:w$, $\sin\beta = h:a$, $\gamma = 2(\varphi - \beta)$, $\alpha = 180^0 - 2\varphi + \beta$, $\beta_1 = 42^0 4' 30'',1$, $\gamma = 123^0 18' 48'',4$, $\alpha = 14^0\ 36'\ 41'',5$, $b = 773$, $c = 964$, $F = 93990$.

24. $\varrho \operatorname{cotg}\frac{1}{2}\alpha = x$, $\varrho\operatorname{cotg}\frac{1}{2}\beta = y$, $\varrho\operatorname{cotg}\frac{1}{2}\gamma = z$; $a = y + z$; $b = x + z$, $c = x + y$, $F = xyz:\varrho$; a) $a = 181396$, $b = 215585$, $c = 278627$, $F = 19548300000$; b) $a = 804,30$, $b = 1006,63$, $c = 864,88$, $F = 334477$.

25. $b = h:\sin\alpha$; $\operatorname{cotg}\frac{1}{2}\gamma = \frac{b}{\varrho} - \operatorname{cotg}\frac{1}{2}\alpha$, $a = h:\sin\beta$; $c = 956$, $a = 533$, $b = 1011$, $\beta = 80^0\ 3'\ 37'',9$, $\gamma = 68^0\ 39'\ 17'',9$, $F = 250950$.

26. $\sin\beta = \frac{h}{a}$; $\operatorname{cotg}\frac{1}{2}\gamma = \frac{a}{\varrho} - \operatorname{cotg}\frac{1}{2}\beta$, $b = h:\sin\alpha$, $\beta_1 = 64^0\ 23'\ 29'',3$, $\gamma = 59^0\ 48'\ 50'',3$, $\alpha = 55^0 47' 40'',4$, $b = 555$, $c = 532$, $F = 122094$.

27. $\operatorname{tang}\frac{1}{2}\gamma = (a - \varrho\operatorname{tang}\frac{1}{2}\beta):\varrho$; $\gamma = 63^0\ 31'\ 8'',3$, $\alpha = 59^0\ 52'\ 46'',3$, $b = 305$, $c = 327$, $F = 43134$.

28. $\operatorname{tang}\frac{1}{2}\gamma = (\varrho_a \operatorname{cotg}\frac{1}{2}\alpha - b):\varrho_a$; $\gamma = 39^0\ 35'\ 51'',9$, $\beta = 13^0\ 41'\ 8'',0$, $a = 1196$, $c = 951$, $F = 134550$.

34. $\cos\alpha = \frac{c^2 + 4(b^2 - m^2)}{4bc}$, $\cos\beta = \frac{3c^2 - 4(b^2 - m^2)}{4ac}$, $a^2 = 2m^2 - b^2 + \frac{1}{2}c^2$; $\alpha = 76^0$, $\beta = 81^0$, $\gamma = 23^0$, $a = 3$, $F = 1,7898$.

35. $\sin\varphi = c\sin\beta:2m$, $a = m\sin(\varphi + \beta):\sin\beta$, $b^2 = m^2 + \frac{1}{4}c^2 - mc\cos(\varphi + \beta)$; $b = 150$, $a = 145$, $\alpha = 73^0\ 44'\ 23'',3$, $\gamma = 9^0\ 31'\ 38'',2$, $F = 1800$.

36. $\operatorname{tang}\alpha = w\sin\frac{1}{2}\gamma:(b - w\cos\frac{1}{2}\gamma)$; $\alpha = 27^0\ 36'\ 1'',3$, $\beta = 100^0\ 14'\ 58'',7$; $a = 19,002$, $c = 32,382$, $F = 302,753$.

37. $\sin\varphi = b \sin\alpha : w$, $\gamma = 2\,(180^0 - \varphi - \alpha)$, $\beta = 2\varphi + \alpha - 180^0$; $a = 24{,}3006$, $c = 81{,}9844$, $\beta = 109^0\,36'\,18'',1$, $\gamma = 56^0\,8'\,41'',8$, $F = 938{,}405$.

41. $b^2 = a^2 + d^2 - 2ad\cos\beta$; $\sin\alpha = a\sin\beta : b$ oder $\operatorname{tang}\alpha = a\sin\beta : (a\cos\beta - d)$; $c = 2a\cos\beta - d$; $b = 195$, $c = 388$, $\alpha = 75^0 44' 59'',5$, $\gamma = 75^0 10' 52'',0$, $F = 36666$.

47. $\cos\frac{1}{2}(\alpha-\beta) = s\sin\frac{1}{2}\gamma : c = \sin(\beta + \frac{1}{2}\gamma)$, $b = c\sin\beta : \sin\gamma$;
a) $\alpha = 11^0\,3'\,18'',3$, $\beta = 3^0\,41'\,42'',8$, $b = 485$, $F = 89094$;
b) $\alpha = 53^0\,51'\,14'',3$, $\beta = 19^0\,8'\,45'',7$, $b = 6330{,}5$, $a = 15586{,}5$.

48. $a = s \,.\, \sin\alpha : [2\cos\frac{1}{2}\gamma \,.\, \sin(\alpha + \frac{1}{2}\gamma)]$, $c = s \,.\, \sin\frac{1}{2}\gamma : \sin(\alpha + \frac{1}{2}\gamma)$, $b = s - a$, oder $a - b = s \,.\, \operatorname{tang}\frac{1}{2}(\alpha - \beta)\operatorname{tang}\frac{1}{2}\gamma$;
a) $a = 12$, $b = 5$, $c = 13$, $F = 30$.
b) $a = 5{,}6951$, $b = 4{,}3049$, $c = 3{,}8346$, $F = 8{,}2512$.

49. $\operatorname{tang}\frac{1}{2}\beta = (s - c)\operatorname{cotg}\frac{1}{2}\alpha : (s + c)$; $\beta = 90^0$, $\gamma = 40^0$, $a = 1$, $b = 1{,}55572$, $F = 0{,}41955$.

50. $\sin\frac{1}{2}\gamma = \frac{c}{s}\cos\frac{1}{2}(\alpha - \beta)$, $a - b = c\sin\frac{1}{2}(\alpha - \beta) : \cos\frac{1}{2}\gamma$; $a = 625$, $b = 509$, $\alpha = 154^0 23' 29'',3$, $\beta = 20^0 36' 34'',9$, $\gamma = 4^0\,59'\,55'',8$, $F = 13860$.

51. $\sin\frac{1}{2}(\alpha - \beta) = d\cos\frac{1}{2}\gamma : c$; $\frac{1}{2}(\alpha + \beta) = 90^0 - \frac{1}{2}\gamma$;
a) $\alpha = 92^0\,4'\,2''$, $\beta = 45^0\,23'\,54''$, $a = 473$, $b = 337$, $F = 53880$; b) $\alpha = 98^0\,1'\,28'',9$, $\beta = 41^0\,58'\,31'',1$, $a = 116{,}961$, $b = 78{,}997$, $F = 2969{,}5$.

52. $\operatorname{cotg}\frac{1}{2}\alpha = (c - d)\operatorname{cotg}\frac{1}{2}\beta : (c + d)$; $\alpha = 79^0\,16'\,41'',8$, $\gamma = 55^0\,43'\,18'',2$, $a = 3{,}5675$, $b = 2{,}5675$, $F = 3{,}7840$.

53. $\operatorname{tang}\frac{1}{2}\beta = (c - d)\operatorname{tang}\frac{1}{2}\alpha : (c + d)$; $\beta = 44^0\,45'\,37'',0$, $\gamma = 57^0\,15'\,27'',9$, $a = 507$, $b = 365$, $F = 77826$.

54. $c = d\cos\frac{1}{2}\gamma : \sin\frac{1}{2}(\beta - \alpha)$; $c = 54{,}8538$, $a = 38{,}2825$, $b = 53{,}6525$, $F = 971{,}225$.

55. $\cos\frac{1}{2}\gamma = c\sin\frac{1}{2}\delta : d$; $\gamma = 139^0 11' 35'',4$, $\alpha = 25^0 24' 12'',3$, $\beta = 15^0\,24'\,12'',3$, $a = 1{,}3129$, $b = 0{,}8129$, $F = 0{,}34872$.

56. $\cos\frac{1}{2}\gamma = s \sin\frac{1}{2}\delta : d$, $a - b = d \sin\frac{1}{2}\gamma : \cos\frac{1}{2}\delta$; $a = 650$, $b = 281$, $c = 861$, $\alpha = 34^0\ 42'\ 29'',0$, $\beta = 14^0\ 15'\ 0'',1$, $\gamma = 131^0\ 2'\ 30'',9$, $F = 68880$.

59. $\sin\frac{1}{2}\gamma = d \cos\frac{1}{2}\delta : f$, $a + b = f \cos\frac{1}{2}\gamma : \sin\frac{1}{2}\delta$; $a = 64,1$, $b = 29,0$, $c = 81,9$, $\alpha = 43^0\ 36'\ 10'',1$, $\beta = 18^0\ 10'\ 50'',0$, $\gamma = 118^0\ 12'\ 59'',9$, $F = 819$.

62. $c = u + v$, $\sin\beta = v \sin\alpha : u$, $a = (u+v)\sin\alpha : \sin\gamma$; $c = 584,0$, $\beta = 17^0\ 13'\ 52'',7$, $\gamma = 90^0\ 15'\ 39'',7$, $a = 557$, $b = 173$, $F = 45953$.

72. $\gamma = \varphi + \psi$; $\operatorname{tang}\frac{1}{2}(\alpha - \beta) = \operatorname{tang}\frac{1}{2}(\psi - \varphi)\operatorname{cotg}\frac{1}{2}\gamma^2$, $c = 2m\sin\varphi : \sin\beta = 2m\sin\psi : \sin\alpha$, $a = 2m\sin\psi : \sin\gamma$, $b = 2m\sin\varphi : \sin\gamma$. **a)** $\alpha = 27^0\,3'\,5'',2$, $\beta = 81^0\,18'\,46'',0$, $\gamma = 71^0\ 38'\ 8'',7$, $a = 72,835$, $b = 158,318$, $c = 152$, $F = 5472$; **b)** $\alpha = \varphi$, $\beta = \psi$, $\gamma = 90^0$, $a = 8$, $b = 6$, $c = 10$, $F = 24$.

73. $c = \sqrt{2(a^2 + b^2 - 2m^2)} = 507$; $\alpha = 36^0\ 38'$, $\beta = 76^0\ 18'$, $\gamma = 67^0\ 4'$, $F = 80904$.

82. $a = \frac{2}{3}\sqrt{m_a{}^2 + 4m_b{}^2 - 4m_a m_b \cos\varphi} = 2$,
$b = \frac{2}{3}\sqrt{4m_a{}^2 + m_b{}^2 - 4m_a m_b \cos\varphi} = 3$,
$c = \frac{2}{3}\sqrt{m_a{}^2 + m_b{}^2 + 2m_a m_b \cos\varphi} = 4$,
$\alpha = 28^0\ 57'\ 18''$, $\beta = 46^0\ 34'\ 4''$, $\gamma = 104^0\ 28'\ 38''$, $F = 2,9047$.

89. $c = \frac{2}{3}h\sin(\psi \pm \varepsilon) : (\sin\psi\sin\varepsilon)$, $\operatorname{tang}\alpha = h : (\frac{1}{2}c \mp h\operatorname{cotg}\psi)$, $\operatorname{tang}\beta = h : (\frac{1}{2}c \pm h\operatorname{cotg}\psi)$; $a = b = 85$, $c = 72$, $\alpha = \beta = 64^0\ 56'\ 32'',6$, $\gamma = 50^0\ 6'\ 54'',8$, $F = 2772$.

90. $d : 4\sin\frac{1}{2}\alpha\sin\frac{1}{2}\beta\cos\frac{1}{2}\gamma = 2r$; $a = 2r\sin\alpha$, u. s. w. **a)** 4,7879; 3,9921; 3,7800; $F = 73,209$; **b)** 43,843; 39,214; 46,556; $F = 796,58$.

91. $b = h : \sin\alpha$, $\operatorname{tang}\frac{1}{2}\beta = h : (2s - h\operatorname{cotg}\frac{1}{2}\alpha)$; $b = 39,163$, $\beta = 80^0\,8'\,23''$, $\gamma = 49^0\ 51'\ 37''$, $a = 30,449$, $c = 30,387$, $F = 455,81$.

94. $\cos\frac{1}{2}(\alpha - \gamma) = \frac{s}{2b\cos\frac{1}{2}\beta}$, $a + c = \frac{s}{\sin\beta}$, $a - c = \frac{b\sin\frac{1}{2}(\alpha-\gamma)}{\cos\frac{1}{2}\beta}$; $a = 123$, $b = 65$, $c = 106$, $\alpha = 88^0 37' 10'',0$, $\beta = 31^0\ 53'\ 26'',8$, $\gamma = 59^0\ 29'\ 23'',2$, $F = 3444$.

102. $b = d : [2\sin\frac{1}{2}(\alpha - \gamma)\cos\frac{1}{2}(\alpha + \gamma)]$; $a = 325$, $b = 85$, $c = 246$, $\beta = 6^0\ 21'\ 34'',8$, $F = 4428$.

103. $a - c = d : \sin\beta$, $a + c = \sqrt{b^2 \sin\frac{1}{2}\beta^2 - \frac{1}{4}d^2} : \sin\frac{1}{2}\beta^2$, $\sin\frac{1}{2}(\alpha - \gamma) = d : (2b \sin\frac{1}{2}\beta)$: $a = 52$, $c = 15$, $\alpha = 130^\circ\, 26'\, 59'',0$, $\gamma = 12^\circ\, 40'\, 49'',4$, $F = 234$.

104. $a - c = d : \sin\beta$, $b^2 = s^2 \sin\frac{1}{2}\beta^2 + \frac{1}{4}d^2 : \sin\frac{1}{2}\beta^2$, $\operatorname{cotg}\frac{1}{2}(\alpha - \gamma) = 2s \sin\frac{1}{2}\beta^2 : d$; $a = 445$, $b = 450$, $c = 85$, $\alpha = 81^\circ\, 12'\, 9'',3$, $\gamma = 10^\circ\, 52'\, 50'',4$, $F = 18900$.

B.

1. $a = 2r \sin\alpha$, $b = 2r \sin\beta$, $c = 2r \sin(\alpha + \beta)$;
a) 1,73206; 1,96962; 1,28558; $F = 1,09643$.
b) 40,368; 53,189; 67,955; $F = 1072,83$.

2. $\sin\alpha = a : 2r$, $b = 2r \sin\beta$; $\alpha_2 = 111^\circ\, 21'\, 44'',6$, $\gamma = 50^\circ\, 41'\, 32'',5$, $b = 221$, $c = 555$, $F = 57114$.

3. $\sin\beta = b : 2r$, $\sin\alpha = h : b$, $a = 2rh : b$, $c = 2r \sin\gamma$; $\beta_1 = 53^\circ\, 7'\, 48'',4$, $\alpha_1 = 14^\circ\, 28'\, 39'',3$, $\gamma = 112^\circ\, 23'\, 32'',3$, $a = 1,250$, $c = 4,623$, $F = 2,3115$.

4. $b = 2r \sin\beta$, $c = s - b$, $\sin\gamma = c : 2r$; $b = 317$, $c = 939$, $\gamma = 13^\circ\, 41'\, 8'',0$ $(166^\circ\, 18'\, 52'',0)$, $\alpha = 161^\circ\, 43'\, 59'',6$ $(9^\circ\, 6'\, 15'',6)$, $a = 1244$ (628), $F = 46650$ (23550).

5. $c = 2r \sin\gamma = 123$, $b = d + c = 365$, $\sin\beta = b : 2r$; $\beta = 12^\circ\, 40'\, 49'',4$ $(167^\circ\, 19'\, 10'',6)$, $\alpha = 163^\circ\, 4'\, 38'',7$ $(8^\circ\, 26'\, 17'',5)$, $a = 484$ (244), $F = 6534$ (3294).

6. $\sin\alpha = a : 2r$, $b - c = a \sin\frac{1}{2}\delta : \cos\frac{1}{2}\alpha$, $b + c = a \cos\frac{1}{2}\delta : \sin\frac{1}{2}\alpha$; $\alpha_1 = 162^\circ\, 51'\, 14'',4$, $\beta = 11^\circ\, 25'\, 16'',3$, $\gamma = 5^\circ\, 43'\, 29'',3$, $b = 401$, $c = 202$, $F = 11940$.

9. $a = 2F : h_a = 11,25$, $b = 2F : h_b = 10$, $\sin\gamma = h_a h_b : 2F$; $c = 9,5688$ $(19,0148)$, $\alpha = 70^\circ\, 8'\, 41'',2$ $(28^\circ\, 15'\, 0'')$, $\beta = 56^\circ\, 43'\, 30'',6$ $(24^\circ\, 52'\, 49'')$, $\gamma = 53^\circ\, 7'\, 48'',2$ $(126^\circ\, 52'\, 11'',8)$.

10. $c = 2F : h = 84$; $p = \frac{1}{2}(c + d)$, $q = \frac{1}{2}(c - d)$, $\operatorname{tang}\alpha = h : q$, $\operatorname{tang}\beta = h : p$, $a = h : \sin\beta$, $b = h : \sin\alpha$; $\alpha = 77^\circ\, 19'\, 10'',6$, $\beta = 28^\circ\, 4'\, 20'',9$, $\gamma = 74^\circ\, 36'\, 28'',5$.

17. $a = \frac{s h_b}{h_a + h_b}$, $b = \frac{s h_a}{h_a + h_b}$, $\sin\gamma = \frac{h_a + h_b}{s}$; $a = 10$; $b = 5{,}0771$, $c = 7{,}7786$, $\alpha = 100^0$, $\beta = 30^0$, $\gamma = 50^0$, $F = 19{,}446$.

19. $\sin\beta = \frac{n}{m}\sin\alpha$; $\beta = 76^0\ 37'\ 20''$, $\gamma = 57^0\ 43'\ 49''$, $a = 77{,}268$, $b = 105{,}126$, $F = 4061{,}45$.

20. $\sin\beta = \frac{n}{m}\sin\alpha$, $a = \frac{mh}{n\sin\alpha}$, $b = \frac{h}{\sin\alpha}$; $a = 19{,}5395$, $b = 23{,}4474$, $c_1 = 41{,}5397$, $\alpha_1 = 13^0\ 33'\ 58'',1$, $\beta_1 = 16^0\ 20'\ 55'',1$, $\gamma_1 = 150^0\ 5'\ 6'',8$, $F = 114{,}242$.

25. $\operatorname{tang}\frac{1}{2}\gamma = \frac{a-b}{a+b}\operatorname{cotg}\frac{1}{2}\delta$; $\gamma = 75^0$, $\alpha = 60^0$, $\beta = 45^0$, $c = 1{,}9268$, $F = 1{,}15$.

26. $\operatorname{tang}\frac{1}{2}(\alpha - \beta) = (m - n)\operatorname{cotg}\frac{1}{2}\gamma : (m + n)$; $\alpha = 74^0\ 36'\ 28'',4$, $\beta = 24^0\ 11'\ 22'',3$, $a = 200$, $b = 85$, $F = 8400$.

27. $\operatorname{tang}\frac{1}{2}\gamma = \frac{m-n}{m+n}\operatorname{cotg}\frac{1}{2}\delta$, $a - b = \frac{c\sin\frac{1}{2}\delta}{\cos\frac{1}{2}\gamma}$, $a + b = \frac{c\cos\frac{1}{2}\delta}{\sin\frac{1}{2}\gamma}$; $a = 51$, $b = 30$, $\alpha = 126^0 52'\ 11'',6$, $\beta = 28^0\ 4'\ 20'',9$, $\gamma = 25^0\ 3'\ 27'',5$, $F = 324$.

28. $\sin\gamma = c : 2r$; $\operatorname{tang}\frac{1}{2}(\alpha - \beta) = (m - n)\operatorname{cotg}\frac{1}{2}\gamma : (m + n)$; $\gamma_1 = 67^0\ 15'\ 12'',5$, $\alpha = 65^0\ 51'\ 28'',3$, $\beta = 46^0 53'\ 19'',1$, $a = 82{,}128$, $b = 65{,}702$, $F = 2022{,}45$.

29. $\alpha = 90^0 - \frac{1}{2}\gamma + \frac{1}{2}\delta$, $a + b = d\operatorname{cotg}\frac{1}{2}\gamma\operatorname{cotg}\frac{1}{2}\delta$; $a = 268$, $b = 281$, $c = 255$, $\alpha = 59^0\ 45'\ 56'',4$, $\beta = 64^0\ 56'\ 32'',6$, $F = 30954$.

30. $\alpha = 90^0 - \frac{1}{2}\gamma + \frac{1}{2}\delta$, $\beta = 90^0 - \frac{1}{2}\gamma - \frac{1}{2}\delta$, $a - b = s\operatorname{tg}\frac{1}{2}\delta\operatorname{tg}\frac{1}{2}\gamma$, $a = 1004$, $b = 305$, $c = 807$, $\alpha = 122^0\ 23'\ 45'',3$, $\beta = 14^0\ 51'\ 46'',2$, $F = 103914$.

31. $a + b = m\sqrt{\operatorname{cotg}\frac{1}{2}\delta\operatorname{cotg}\frac{1}{2}\gamma}$, $a - b = m\sqrt{\operatorname{tg}\frac{1}{2}\delta\operatorname{tg}\frac{1}{2}\gamma}$; $a = 428$, $b = 257$, $c = 471$, $\alpha = 64^0\ 22'\ 24'',9$, $\beta = 32^0\ 46'\ 44'',7$, $F = 54570$.

32. $\alpha = 21^0$, $\beta = 14^0$, $\gamma = 145^0$, $a = 832{,}2$, $b = 561{,}8$, $c = 1331{,}97$, $F = 134082$.

33. $\alpha = 35^0\ 39'\ 33'',3$, $\beta = 71^0\ 19'\ 6'',6$, $\gamma = 73^0 1'\ 20'',1$, $c : a : b = 0{,}95640 : 0{,}58296 : 0{,}9473 = 13{,}1247\cdots : 8 : 13$.

34. $a = 2r \sin\alpha$, $\cos\frac{1}{2}(\beta - \gamma) = s : 4r\cos\frac{1}{2}\alpha$; $a = 40$, $b = 37$, $c = 13$, $\beta = 67^0\ 22'\ 48'',5$, $\gamma = 18^0\ 55'\ 28'',7$, $F = 240$.

35. $\sin\alpha = a : 2r$. Vergl. 34. $\alpha = 52^0\ 27'\ 13'',3$, $\beta = 105^0\ 38'\ 40'',0$, $\gamma = 21^0\ 54'\ 6'',7$, $b = 37{,}286$, $c = 14{,}443$, $F = 213{,}49$.

36. $\sin\gamma = c : 2r$, $\sin\frac{1}{2}(\alpha - \beta) = d\cos\frac{1}{2}\gamma : c = d : (4r\sin\frac{1}{2}\gamma)$, $a + b = c\cos\frac{1}{2}(\alpha - \beta) : \sin\frac{1}{2}\gamma$; $a = 714$ $(357{,}09)$, $b = 365$ $(8{,}09)$, $\alpha = 155^0\ 57'\ 50'',2$ $(11^0\ 45'\ 13'',5)$, $\beta = 12^0\,1'\,4'',9\,(0^0\,15'\,51'',5)$, $\gamma = 12^0\,1'\,4'',9\,(167^0\,58'\,55'')$, $F = 27132$ $(300{,}76)$.

37. $a = 2r\sin\alpha$, $b + c = 2(s - r\sin\alpha)$. Vergl. 34. $a = 101$, $b = 29$, $c = 120$, $\beta = 11^0\ 25'\ 16'',3$, $\gamma = 124^0\ 58'\ 33'',6$, $F = 1200$.

38. $\sin\gamma = v\sin\beta$; $2r = s : (2\cos\frac{1}{2}\alpha\cos\frac{1}{2}\beta\cos\frac{1}{2}\gamma)$, $a = 2r\sin\alpha$, $b = 2r\sin\beta$, $c = 2r\sin\gamma$; $a = 85$, $b = 400$, $c = 325$, $\alpha = 6^0\,21'\,34'',8$, $\gamma = 25^0\,3'\,27'',4$, $F = 7200$.

42. $2r = f : \sqrt{\sin\beta^2 - \sin\gamma^2} = f : \sqrt{\sin\alpha\sin(\beta - \gamma)}$; $a = 2r\sin\alpha$, etc.; $b - c = d\sqrt{\operatorname{tg}\frac{1}{2}\alpha\operatorname{tg}\frac{1}{2}(\beta - \gamma)}$, $b + c = d\sqrt{\operatorname{cotg}\frac{1}{2}\alpha\operatorname{cotg}\frac{1}{2}(\beta - \gamma)}$;

a) $a = 229$, $b = 232$, $c = 61$, $\alpha = 79^0\ 36'\ 40''$, $F = 6960$.

b) $a = 52$, $b = 53$, $c = 51$, $\alpha = 59^0\ 57'\ 47'',7$, $F = 1170$.

43. $\sin(\beta - \gamma) = \frac{f^2\sin\alpha}{a^2}$, $b - c = \frac{a\sin\frac{1}{2}(\beta - \gamma)}{\cos\frac{1}{2}\alpha}$, $b + c = \frac{a\cos\frac{1}{2}(\beta - \gamma)}{\sin\frac{1}{2}\alpha} = \frac{d^2}{b - c}$; $b = 80$, $c = 61$, $\beta_1 = 46^0\ 12'\ 45'',4$, $\gamma = 79^0\ 36'\ 40'',0$, $F = 2400$.

44. $\sin\alpha = \frac{d^2\sin\delta}{f^2}$, $b - c = \frac{a\sin\frac{1}{2}\delta}{\cos\frac{1}{2}\alpha}$, $b + c = \frac{f^2}{b - c} = \frac{a\cos\frac{1}{2}\delta}{\sin\frac{1}{2}\alpha}$, $F = \frac{1}{2}f^2 \cdot \frac{\sin\beta\sin\gamma}{\sin\delta}$; $\alpha = 41^0\,52'\,4'',3\,(138^0\,7'\,55'',7)$, $\beta = 87^0\,12'\,20'',0\,(39^0\,4'\,24'',3)$, $\gamma = 50^0\ 55'\ 35'',7$ $(2^0\ 47'\ 40'',0)$, $b = 1082$ $(682{,}81)$, $c = 841$ $(52{,}81)$, $F = 179679$ $(7120{,}83)$.

45. $(b+c)^2 = s^2 + \frac{s^2-a^2}{\cos\alpha}$, $(b-c)^2 = s^2 - \frac{s^2-a^2}{\cos\alpha}$,
$\cos(\beta-\gamma) = \frac{s^2\sin\alpha^2 - a^2}{a^2\cos\alpha}$; $b = 40$, $c = 13$,
$\beta = 93^0\, 41'\, 42'',8$, $\gamma = 18^0\, 55'\, 28'',7$, $F = 240$.

46. $\cos(\beta-\gamma) = \frac{2p^2}{a^2}\sin\alpha^2 - \cos\alpha$;
$(b+c)^2 = a^2 + 4p^2\cos\tfrac{1}{2}\alpha^2$, $(b-c)^2 = a^2 - 4p^2\sin\tfrac{1}{2}\alpha^2$;
$b = 241$, $c = 182$, $\beta = 17^0\, 3'\, 42''$, $\gamma = 12^0\, 48'\, 4''$,
$F = 10920$.

47. $a^2 = \tfrac{1}{2}q^2 + p^2 - 2p^2\cos\tfrac{1}{2}\alpha^2 = \tfrac{1}{2}q^2 - p^2 + 2p^2\sin\tfrac{1}{2}\alpha^2$,
$(b+c)^2 = \tfrac{1}{2}q^2 + p^2 + 2p^2\cos\tfrac{1}{2}\alpha^2$,
$(b-c)^2 = \tfrac{1}{2}q^2 - p^2 - 2p^2\sin\tfrac{1}{2}\alpha^2$; $a = 5$, $b = 7$,
$c = 6$, $\alpha = 44^0\, 24'\, 54''$, $\beta = 78^0\, 27'\, 48''$, $\gamma = 57^0\, 7'\, 18''$,
$F = 14{,}697$.

48. $a + b - c = 4p^2\sin\tfrac{1}{2}\alpha^2 : d$; $a = 1676$, $b = 425$,
$c = 1263$, $\beta = 3^0\, 56'\, 59'',6$, $\gamma = 11^0\, 48'\, 44'',2$,
$F = 72906$.

49. $a + b + c = 4p^2\cos\tfrac{1}{2}\alpha^2 : d$; $a = 572$, $b = 293$,
$c = 579$, $\beta = 29^0\, 29'\, 13'',6$, $\gamma = 76^0\, 34'\, 49'',2$,
$F = 81510$.

50. $b + c - a = 2p^2\cos\tfrac{1}{2}\alpha^2 : s$; $a = 51{,}5$, $b = 31$,
$c = 24{,}5$, $\beta = 24^0\, 45'\, 40''$, $\gamma = 19^0\, 19'\, 48''$, $F = 264{,}23$.

51. $c = \frac{s}{2} + \sqrt{\frac{s^2}{4} - \frac{2F}{\sin\alpha}}$, $b = \frac{s}{2} - \sqrt{\frac{s^2}{4} - \frac{2F}{\sin\alpha}}$,
$a^2 = s^2 - 4F\operatorname{cotg}\tfrac{1}{2}\alpha$; $c = 19{,}0319$, $b = 10{,}9681$,
$a = 10{,}7725$, $\beta = 29^0\, 11'\, 31''$, $\gamma = 122^0\, 11'\, 4''$,
$F = 1250$.

52. $c^2 = d^2 + 4F\operatorname{tang}\tfrac{1}{2}\gamma$, $(a+b)^2 = d^2 + 8F : \sin\gamma$;
$a = 1077$, $b = 724$, $c = 365$, $\alpha = 161^0\, 57'\, 23'',0$,
$\beta = 12^0\, 1'\, 4'',9$.

53. $(b+c)^2 = f^2 + 4F : \sin\alpha$, $(b-c)^2 = f^2 - 4F : \sin\alpha$;
$a = 724$, $b = 545$, $c = 183$, $\beta = 10^0\, 23'\, 20'',0$,
$\gamma = 3^0\, 28'\, 17'',1$.

54. $\operatorname{cotg}\alpha = (f^2 - a^2) : 4F$, $(b+c)^2 = f^2 + \sqrt{16F^2 + (f-a)^2}$,
$(b-c)^2 = f^2 - \sqrt{16F^2 + (f-a)^2}$, $\sin\gamma = 2F : ab$,
$\sin\beta = 2F : ac$; $b = 15$, $c = 13$, $\alpha = 59^0\, 29'\, 23'',1$,
$\beta = 67^0\, 22'\, 48'',5$, $\gamma = 53^0\, 7'\, 48'',4$.

55. $c=\sqrt{2Fv}$, $\sin\gamma=2F:p^2$, $(a+b)^2=2Fv+4p^2\cos\frac{1}{2}\gamma^2$, $(a-b)^2=2Fv-4p^2\sin\frac{1}{2}\gamma^2$, $\sin\beta=h:a$, $\sin\alpha=h:b$; $a=37$, $b=13$, $c=40$, $\alpha=67^0\,22'\,48'',5$, $\beta=18^0\,55'\,28'',7$, $\gamma=93^0\,41'\,42'',8$.

56. $\sin\gamma=ch:p^2$. Vergl. 46. $a=b=c=9$, $\alpha=\beta=\gamma=60^0$.

57. $(a+b)^2=c^2+2ch\operatorname{cotg}\frac{1}{2}\gamma$, $(a-b)^2=c^2-2ch\operatorname{tg}\frac{1}{2}\gamma$, $\cos(\alpha-\beta)=\frac{2h}{c}\sin\gamma-\cos\gamma$; $a=241$, $b=169$, $\alpha=134^0\,45'\,34''$, $\beta=29^0\,51'\,49''$, $F=5400$.

58. $\sin\alpha=a:2r$, $\cos(\beta-\gamma)=\frac{2F}{ar}-\cos\alpha$, $(b+c)^2=a^2+\frac{16rF}{a}\cos\frac{1}{2}\alpha^2$, $(b-c)^2=a^2-\frac{16rF}{a}\sin\frac{1}{2}\alpha^2$; $\alpha=91^0\,22'\,50'',0$, $\beta=35^0\,29'\,21'',6$, $\gamma=53^0\,7'\,48'',4$, $b=500$, $c=689$.

59. $\operatorname{cotg}\frac{1}{2}\gamma=(s^2-c^2):4F$, $(a-b)^2=s^2-(8F:\sin\gamma)$, $\cos\frac{1}{2}(\alpha-\beta)=\frac{s}{c}\sin\frac{1}{2}\gamma$; $\alpha=119^0\,16'\,42''$, $\beta=32^0\,38'\,58''$, $\gamma=28^0\,4'\,20''$, $a=18{,}5356$, $b=11{,}4644$, oder $\alpha=20^0\,36',6$, $\beta=5^0\,12',3$, $\gamma=154^0\,11',1$, $a=485$, $b=125$.

60. $a=s-\frac{F}{s}\operatorname{cotg}\frac{1}{2}\alpha$, $b+c=s+\frac{F}{s}\operatorname{cotg}\frac{1}{2}\alpha$, $bc=2F:\sin\alpha$; $a=1772$, $b=593$, $c=1479$, $\beta=18^0\,19'\,28'',9$, $\gamma=51^0\,38'\,31'',2$.

61. $a=(F\operatorname{cotg}\frac{1}{2}\alpha-d^2):d$, $b+c=2d+a$, $bc=2F:\sin\alpha$; $a=624$, $b=205$, $c=445$, $\beta=10^0\,52'\,50'',4$, $\gamma=24^0\,11'\,22'',3$.

62. $c=\sqrt{s^2+h^2\operatorname{cotg}\frac{1}{2}\gamma^2}-h\operatorname{cotg}\frac{1}{2}\gamma$, $(a-b)^2=s^2-4ch:\sin\gamma$, $s\cos\frac{1}{2}(\alpha-\beta)^2-2h\cos\frac{1}{2}\gamma\cos\frac{1}{2}(\alpha-\beta)=s\sin\frac{1}{2}\gamma^2$; $a=6{,}5$, $b=7{,}5$, $c=7{,}0$, $\alpha=53^0\,7'\,48''$, $\beta=67^0\,22'\,48''$, $F=21$.

63. $c=\sqrt{d^2+h^2\operatorname{tang}\frac{1}{2}\gamma^2}+h\operatorname{tang}\frac{1}{2}\gamma$, $(a+b)^2=d^2+4ch:\sin\gamma$, $\sin\frac{1}{2}(\alpha-\beta)=d\cos\frac{1}{2}\gamma:c$; $c=150$, $a=197$, $b=53$, $\alpha=148^0\,6'\,33'',2$, $\beta=8^0\,10'\,16'',4$, $F=2100$.

64. $c = 2s^2 : (2s + h \operatorname{cotg} \frac{1}{2}\gamma)$, $a + b = 2s - c$,
$(a-b)^2 = c^2 - 2rh \operatorname{tg} \frac{1}{2}\gamma$, $\cos \frac{1}{2}(\alpha - \beta) = \frac{h}{s} \cos \frac{1}{2}\gamma + \sin \frac{1}{2}\gamma$;
$a = 305$, $b = 785$, $c = 872$, $\alpha = 20^0\, 21'\, 3'',7$,
$\beta = 63^0\, 31'\, 8'',3$, $F = 119028$.

65. $c = 2r \sin \gamma$, $(a + b)^2 = 4r^2 \sin \gamma^2 + 8rh \cos \frac{1}{2}\gamma^2$,
$(a - b)^2 = 4r^2 \sin \gamma^2 - 8rh \cos \frac{1}{2}\gamma^2$,
$\cos (\alpha - \beta) = \frac{h}{r} - \cos \gamma$; $a = 629$, $b = 821$,
$c = 1160$, $\alpha = 31^0\, 30'\, 8'',5$, $\beta = 43^0\, 0'\, 10'',3$,
$F = 248820$.

66. $\sin \alpha = h : b$, $c = (b^2 - d^2) : 2(d \pm \sqrt{b^2 - h^2})$,
$a = c + d$, $\sin \beta = h : a$, $a + c = b(b + d \cos \alpha) : (d + b \cos \alpha)$;
$a_1 = 365$, $c = 363$, $\alpha_1 = 6^0\, 1'\, 32'',1$,
$\beta = 167^0\, 58'\, 55'',1$, $\gamma = 5^0\, 59'\, 32'',8$, $F = 13794$.

67. $\sin \beta = h : a$, $c = \frac{1}{2}(\sqrt{a^2 - h^2} \pm \sqrt{2s^2 - a^2 - h^2})$,
$b^2 = s^2 - c^2$, $\sin \alpha = h : b$; $b = 65$ $(36,674)$,
$c = 33$ (63), $\alpha = 14^0\, 15'\, 0'',1$ $(25^0\, 52'\, 0'',0)$,
$\beta = 151^0\, 55'\, 39'',1$ $(28^0\, 4'\, 20'',9)$, $\gamma = 13^0\, 49'\, 20'',8$,
$(126^0\, 3'\, 39'',1)$, $F = 264$ (504).

68. $\sin \beta = h : a$, $c = (a^2 + d^2) : 2a \cos \beta$, $b^2 = c^2 - d^2$,
$\sin \alpha = h : b$; $\beta_1 = 12^0\, 40'\, 49'',4$, $\alpha = 143^0\, 7'\, 48'',4$,
$\gamma = 24^0\, 11'\, 22'',2$, $b = 15$, $c = 28$, $F = 126$.

69. $\operatorname{cotg} \alpha = \frac{16F^2 + d^4 - a^4}{8a^2F}$, $b^2 = \frac{16F^2 + (a^2 + d^2)^2}{4a^2}$,
$c^2 = \frac{16F^2 + (a^2 - d^2)^2}{4a^2}$; $b = 26$, $c = 25$, $\alpha = 6^0\, 21'\, 34'',8$,
$\beta = 106^0\, 15'\, 36'',7$, $\gamma = 67^0\, 22'\, 48'',5$.

70. $c = \sqrt{2s^2 \operatorname{tg} \frac{1}{2}\gamma + \frac{1}{4}(2s \operatorname{tg} \frac{1}{2}\gamma - d)^2} - s \operatorname{tg} \frac{1}{2}\gamma + \frac{1}{2}d$,
$h = c - d$, $a + b = 2s - c$, $ab = ch : \sin \gamma$;
$a = 241$, $b = 169$, $c = 328$, $\alpha = 45^0\, 14'\, 23''$,
$\beta = 29^0\, 51'\, 46''$, $F = 19680$.

71. $b = \frac{s_1 + s \cos \alpha \pm \sqrt{4 s_1 \cos \frac{1}{2}\alpha^2 (s_1 - s) + s^2 \cos \alpha^2}}{1 + 2 \cos \alpha}$, etc.,
$a = 764$, $b = 485$, $c = 867$, $\alpha = 61^0\, 21'\, 0'',3$,
$\beta = 33^0\, 51'\, 18'',1$, $\gamma = 84^0\, 47'\, 41'',6$, $F = 184506$.

72. $c = \{2(s+s_1)\sin\frac{1}{2}\gamma^2 \pm \sqrt{(s-s_1)^2\cos\gamma^2 + 4ss_1\sin\frac{1}{2}\gamma^2}\}:$ $(2\sin\frac{1}{2}\gamma^2 - \cos\gamma)$, etc., $a = 1532$, $b = 533$, $c = 1299$, $\alpha = 105^0\ 44'\ 7'',6$, $\beta = 19^0\ 33'\ 53'',3$, $F = 333210$.

73. $c^2\cos\frac{1}{2}\alpha^2 - c\,[(2s-d)\cos\frac{1}{2}\alpha^2 - s] = sd$, etc.; $a = 964$, $b = 773$, $c = 291$, $\beta = 42^0\ 4'\ 30'',1$, $\gamma = 14^0\ 36'\ 41'',5$, $F = 93990$.

74. $c = \sqrt{d^2 + 4s^2\operatorname{tg}\frac{1}{2}\gamma^2 : \cos\frac{1}{2}\gamma^2} - 2s\operatorname{tg}\frac{1}{2}\gamma^2$; $a = 956$, $b = 533$, $c = 1011$, $\alpha = 68^0\ 39'\ 17'',9$, $\beta = 31^0\ 17'\ 4'',2$, $F = 250950$.

75. $c^2\cos\frac{1}{2}\beta^2 - c\,(s\cos\beta + d\cos\frac{1}{2}\beta^2) + sd = 0$; $a = 604$, $b = 545$, $c = 663$, $\alpha = 59^0\ 2'\ 21'',3$, $\gamma = 70^0\ 16'\ 6'',2$, $F = 154926$.

80. $\sin\beta = h : a$, $c = (s^2 - a^2) : 2(s - \sqrt{a^2 - h^2})$, $b = s - c$, $\operatorname{tang}(\beta + \frac{1}{2}\alpha) = s\sin\beta : (s\cos\beta - a)$; $\beta_1 = 14^0\ 36'\ 41'',5$, $b = 197$, $c = 776$, $\alpha = 81^0\ 49'\ 43'',6$, $\gamma = 83^0\ 33'\ 34'',9$, $F = 75660$.

81. $a = h : \sin\beta$, $c = (s^2 - a^2) : 2(s - h\operatorname{cotg}\beta)$, $b = s - c$; $a = 569$, $b = 255$, $c = 628$, $\alpha = 64^0\ 56'\ 32'',6$, $\gamma = 91^0\ 6'\ 19'',3$, $F = 72534$.

82. $\operatorname{tang}\frac{1}{2}\alpha = 2\varrho : (s-a)$, $b - c = \sqrt{a^2 - s^2\sin\frac{1}{2}\alpha^2} : \cos\frac{1}{2}\alpha$, $\cos\frac{1}{2}(\beta - \gamma) = s\sin\frac{1}{2}\alpha : a$; $b = 15$, $c = 13$, $\alpha = 59^0\ 29'\ 23'',1$, $\beta = 67^0\ 22'\ 48'',5$, $\gamma = 53^0\ 7'\ 48'',4$, $F = 84$.

83. $a = s - 2\varrho\cot\frac{1}{2}\alpha$. Vergl. 82. $a = 364$, $b = 425$, $c = 303$, $\beta = 78^0\ 34'\ 43'',7$, $\gamma = 44^0\ 19'\ 57'',7$, $F = 54054$.

84. $\operatorname{tang}\frac{1}{2}\beta = 2\varrho : (a - d)$, $\operatorname{tang}\frac{1}{2}\gamma = 2\varrho : (s - c)$; $a = 1200$, $b = 1201$, $c = 49$, $\alpha = 87^0\ 39'\ 42'',2$, $\beta = 90^0$, $\gamma = 2^0\ 20'\ 17'',8$, $F = 29400$.

89. $\operatorname{tang}\frac{1}{2}\gamma = 2\varrho^2 : c\,(h - 2\varrho)$, $a + b = c\,(h - \varrho) : \varrho$, $(a - b)^2 = c^2 - 4\varrho^2 h : (h - 2\varrho)$; $a = 881$, $b = 1700$, $\alpha = 28^0\ 4'\ 20'',9$, $\beta = 65^0\ 14'\ 18'',6$, $\gamma = 86^0\ 41'\ 20'',5$, $F = 747600$.

90. $c = 2\varrho^2\operatorname{cotg}\frac{1}{2}\gamma : (h - 2\varrho)$. Vergl. 89. $\cos\frac{1}{2}(\alpha - \beta) = (h - \varrho)\sin\frac{1}{2}\gamma : \varrho$; $a = 1201$, $b = 1300$, $c = 549$, $\alpha = 67^0\ 22'\ 48'',5$, $\beta = 87^0\ 39'\ 42'',2$, $F = 329400$.

91. $a = s - \varrho \operatorname{cotg} \frac{1}{2}\alpha$, $b + c = s + \varrho \operatorname{cotg} \frac{1}{2}\alpha$, $bc = 2\varrho s : \sin\alpha$; $a = 13$, $b = 14$, $c = 15$, $\beta = 59^0\, 29'\, 23''$, $\gamma = 67^0\, 22'\, 49''$, $F = 84$.

92. $\operatorname{tang} \frac{1}{2}\gamma = \varrho : d$, $b = d + \varrho \cot \frac{1}{2}\alpha$, $a = d + \varrho \operatorname{cotg} \frac{1}{2}\beta$, $c = \varrho\, (\operatorname{cotg} \frac{1}{2}\alpha + \operatorname{cotg} \frac{1}{2}\beta)$, $a = 601$, $b = 969$, $c = 482$, $\gamma = 23^0\, 32'\, 11'',4$, $\beta = 29^0\, 51'\, 46'',3$, $F = 116280$.

93. $b - c = \sqrt{a^2 - 4\varrho^2 - 4a\varrho \operatorname{tg} \frac{1}{2}\alpha}$,
$\cos \frac{1}{2}(\beta - \gamma) = \frac{2\varrho}{a} \cos \frac{1}{2}\alpha + \sin \frac{1}{2}\alpha$;

a) $b = 3,75$, $c = 3,50$, $\beta = 67^0\, 22'\, 49''$, $\gamma = 59^0\, 29'\, 23''$, $F = 10,25$;

b) $b = 510$, $c = 317$, $\beta = 68^0 23'\, 7'',1$, $\gamma = 35^0 18'\, 0'',9$, $F = 78540$.

94. $b + c = 2\varrho_a \operatorname{ctg} \frac{1}{2}\alpha - a$, $b - c = \sqrt{a^2 - 4\varrho_a{}^2 + 4a\varrho_a \operatorname{tg} \frac{1}{2}\alpha}$,
$\cos \frac{1}{2}(\beta - \gamma) = \frac{2\varrho_a}{a} \cos \frac{1}{2}\alpha - \sin \frac{1}{2}\alpha$; $b = 209$, $c = 241$, $\beta = 60^0\, 8'\, 14''$, $\gamma = 90^0$, $F = 12540$.

95. $a - c = 2\varrho_a \operatorname{tg} \frac{1}{2}\beta - b$, $(a + c)^2 = b^2 - 4\varrho_a{}^2 + 4b\varrho_a \operatorname{ctg} \frac{1}{2}\beta$,
$\sin \frac{1}{2}(\alpha - \gamma) = \frac{2\varrho_a}{b} \sin \frac{1}{2}\beta - \cos \frac{1}{2}\beta$; $a = 1040$, $c = 53$, $\alpha = 119^0\, 20'\, 41'',0$, $\gamma = 2^0\, 32'\, 45'',8$, $F = 23400$.

96. $\operatorname{cotg} \frac{1}{2}\gamma = \frac{a}{\varrho} - \operatorname{cotg} \frac{1}{2}\beta$, $c = \varrho\, (\operatorname{cotg} \frac{1}{2}\alpha + \operatorname{cotg} \frac{1}{2}\beta)$,
$b = a - \frac{\varrho \sin \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta}$; $\gamma = 30^0\, 30'\, 36'',8$, $\alpha = 70^0\, 54'\, 39'',5$, $b = 195$, $c = 101$, $F = 9306$.

97. $\sin\alpha = a : 2r$, $\cos \frac{1}{2}(\beta - \gamma) = (a + 2\varrho \operatorname{cotg} \frac{1}{2}\alpha) : 4r \cos \frac{1}{2}\alpha$; $\alpha_1 = 119^0\, 20'\, 41'',0$, $\beta = 58^0\, 6'\, 33'',2$, $\gamma = 2^0\, 32'\, 45'',8$, $b = 1013$, $c = 53$, $F = 23400$.

98. $a = 2r \sin\alpha$. Vergl. 97. $a = 154,86$, $b = 167,32$, $c = 108,69$, $\beta = 76^0\, 36'\, 35''$, $\gamma = 39^0\, 11'\, 25''$, $F = 8186,8$.

99. $b + c = a\, (\varrho_a + \varrho) : (\varrho_a - \varrho)$, $b - c = \sqrt{a^2 - 4\varrho\varrho_a}$, $\operatorname{tg} \frac{1}{2}\alpha = (\varrho_a - \varrho) : a$, $\cos \frac{1}{2}(\beta - \gamma) = (\varrho_a + \varrho) \cos \frac{1}{2}\alpha : a$; $b = 58$, $c = 41$, $\alpha = 59^0\, 4'\, 39'',3$, $\beta = 77^0\, 19'\, 10'',6$, $\gamma = 43^0\, 36'\, 10'',1$, $F = 1020$.

100. $c = s\,(\varrho_c - \varrho) : (\varrho_c + \varrho)$, $\operatorname{tg} \frac{1}{2}\gamma = (\varrho_c + \varrho) : s$, $(a - b)^2 = c^2 - 4\,\varrho\,\varrho_c$; $a = 2732$, $b = 689$, $c = 2055$, $\alpha = 167^0\ 37'\ 57'',9$, $\beta = 3^0\ 5'\ 46'',7$, $\gamma = 9^0\ 16'\ 15'',4$, $F = 151626$.

104. $b^2 = a^2 + \frac{4}{3}(m_a^2 - m_b^2)$, $c^2 = \frac{2}{3}(m_a^2 + 2m_b^2) - \frac{1}{2}a^2$; $b = 15$, $c = 13$, $\alpha = 59^0\ 29'\ 23'',1$, $\beta = 67^0\ 22'\ 48'',5$, $\gamma = 53^0\ 7'\ 48'',4$, $F = 84$.

105. $b - c = \sqrt{4m^2 + a^2 - s^2}$; $a = b = c = 1{,}73205$, $\alpha = \beta = \gamma = 60^0$, $F = 1{,}29904$.

106. $b + c = \sqrt{4m^2 + a^2 - d^2}$; $b = c = 100$, $\alpha = 160^0$, $\beta = \gamma = 10^0$, $F = 868{,}24$.

107. $b^2 = 2f^2 - 4m_b^2$, $a^2 = 2f^2 - \frac{4}{3}(2m_b^2 + m_a^2)$, $c^2 = f^2 - a^2$; $a = b = c = 1$, $\alpha = \beta = \gamma = 60^0$, $F = \frac{1}{4}\sqrt{3} = 0{,}43301$.

108. $c^2 = m_a^2 + m_b^2 - \frac{1}{4}f^2$, $a^2 - b^2 = \frac{4}{3}(m_b^2 - m_a^2)$; $c = 5$, $a = 4$, $b = 3$, $\alpha = 53^0\ 7'\ 48'',4$, $\beta = 36^0\ 52'\ 11'',6$, $\gamma = 90^0$, $F = 6$.

109. $a = \frac{2}{3}\sqrt{2m_b^2 + 2m_c^2 - m_a^2}$, $b = \frac{2}{3}\sqrt{2m_a^2 + 2m_c^2 - m_b^2}$, $c = \frac{2}{3}\sqrt{2m_a^2 + 2m_b^2 - m_c^2}$; $a = 2$, $b = 3{,}46410$, $c = 4$, $\alpha = 30^0$, $\beta = 60^0$, $\gamma = 90^0$, $F = 3{,}46410$.

110. $b^2 = \frac{2}{3}(f^2 + a^2 - 2m^2)$, $c^2 = \frac{1}{3}(f^2 + 4m^2 - 2a^2)$, $\cos\alpha = (f^2 - a^2) : 2bc$, $\cos\beta = (8m^2 - f^2 - a^2) : 6ac$, $\cos\gamma = (7a^2 + f^2 - 8m^2) : 6ab$; $b = 56{,}2$, $c = 46{,}2$, $\alpha = 34^0\ 42'\ 29''$, $\beta = 55^0\ 17'\ 31''$, $F = 739{,}2$.

111. $c^2 = 4m^2 + d^2 - 2a^2$, $b^2 = 2(2m^2 + d^2 - a^2)$, $\cos\beta = (a^2 - d^2) : 2ac$, $\cos\gamma = (a^2 + d^2) : 2ab$; $b = 208$, $c = 185$, $\alpha = \gamma = 55^0\ 47'\ 40'',4$, $\beta = 68^0\ 24'\ 39'',2$, $F = 15912$.

112. $(b + c)^2 = (4m^2 \cos\frac{1}{2}\alpha^2 - a^2 \sin\frac{1}{2}\alpha^2) : \cos\alpha$, $(b - c)^2 = (a^2 \cos\frac{1}{2}\alpha^2 - 4m^2 \sin\frac{1}{2}\alpha^2) : \cos\alpha$, $\operatorname{tg}\frac{1}{2}(\beta - \gamma) = \sqrt{(a^2 \operatorname{cotg}\frac{1}{2}\alpha^2 - 4m^2) : (4m^2 - a^2 \operatorname{tg}\frac{1}{2}\alpha^2)}$; $b = c = 317$, $\beta = \gamma = 76^0\ 18'\ 52''$, $F = 23100$.

113. $a = \sqrt{4m^2 + 4h^2 \operatorname{cotg} \alpha^2} - 2h \operatorname{cotg} \alpha$;
$(b + c)^2 = 2m^2 + \frac{1}{2}a^2 + 2ah : \sin \alpha$,
$(b - c)^2 = 2m^2 + \frac{1}{2}a^2 - 2ah : \sin \alpha$;
$a = 508$, $b = 401$, $c = 615$, $\beta = 40^0\ 26'\ 59'',0$,
$\gamma = 84^0\ 16'\ 30'',7$, $F = 101346$.

114. $\operatorname{tg} \beta = \frac{2m \cos \varphi}{a - 2m \sin \varphi}$, $\operatorname{tg} \gamma = \frac{2m \cos \varphi}{a + 2m \sin \varphi}$,
$b = \frac{1}{2}\sqrt{a^2 + 4am \sin \varphi + 4m^2}$,
$c = \frac{1}{2}\sqrt{a^2 - 4am \sin \varphi + 4m^2}$;
$\beta = 67^0\ 2'\ 12'',1$, $\gamma = 50^0\ 33'\ 55'',3$, $\alpha = 62^0 23' 52'',6$,
$b = 37{,}742$, $c = 31{,}659$, $F = 529{,}42$.

117. $a = us : (u + v)$, $b = vs : (u + v)$, $c = u + v$;
$a = 617$, $b = 233$, $c = 816$, $\alpha = 26^0\ 47'\ 6'',0$,
$\beta = 9^0\ 57'\ 53'',5$, $\gamma = 143^0\ 25'\ 0'',5$, $F = 42840$.

118. $\operatorname{tang} \frac{1}{2}(\alpha - \beta) = (u - v) \operatorname{cotg} \frac{1}{2}\gamma : (u + v)$, $c = u + v$;
$a = 617$, $b = 137$, $c = 696$, $\alpha = 50^0\ 2'\ 1'',6$,
$\beta = 9^0\ 57'\ 53'',5$, $F = 36540$.

119. $c = u + v$, $\sin \alpha = u \sin \beta : v$; $a_1 = 173$, $b_1 = 557$,
$c_1 = 584$, $\alpha_1 = 17^0\ 13'\ 52'',7$, $\gamma_1 = 90^0\ 15'\ 39'',7$,
$F_1 = 48180$.

120. $c^2 = \frac{(ab - w^2)(a + b)^2}{ab}$, $\cos \frac{1}{2}\gamma = \frac{(a + b) w}{2ab}$;
$c = 927{,}18$, $\beta = 52^0$, $\alpha = 60^0$, $\gamma = 68^0$, $F = 316364$.

121. $b = \sqrt{\frac{n}{m} w^2 + \frac{c^2 n^2}{(m + n)^2}}$, $a = \sqrt{\frac{m}{n} w^2 + \frac{c^2 m^2}{(m + n)^2}}$,
$\sin \frac{1}{2}(\beta - \alpha) = \frac{w(n^2 - m^2)}{2cmn}$, $\operatorname{cotg} \frac{1}{2}\gamma = \frac{n + m}{n - m} \operatorname{tang} \frac{1}{2}(\alpha - \beta)$;
$a = 14$, $b = 50$, $\alpha = 16^0\ 15'\ 36'',7$, $\beta = 90^0$,
$\gamma = 73^0\ 44'\ 23'',3$, $F = 336$.

122. $\cos \frac{1}{2}(\beta - \alpha) = \frac{\sin \frac{1}{2}\gamma}{c} (w \cos \frac{1}{2}\gamma \pm \sqrt{c^2 + w^2 \cos \frac{1}{2}\gamma^2})$,
$\alpha = \beta = 58^0\ 6'\ 33'',2$, $a = b = 53$, $F = 1260$.

123. $a = \sqrt{s^2 - 2sw \cos \frac{1}{2}\gamma}$, $b - c = \sqrt{s^2 - 2sw : \cos \frac{1}{2}\gamma}$,
$\cos \frac{1}{2}(\alpha - \beta)^2 = s \sin \frac{1}{2}\gamma^2 : (s - 2w \cos \frac{1}{2}\gamma)$;
$c = 558$, $b = a = 521$, $\beta = \alpha = 57^0\ 37'\ 17'',7$,
$F = 122760$.

124. $\cos \frac{1}{2}\gamma = \frac{s^2 - c^2}{2sw}$, $(a - b)^2 = s^2 - \frac{4s^2w^2}{s^2 - c^2}$;
$a = b = 293$, $\alpha = \beta = 76^0\ 34'\ 49'',2$,
$\gamma = 26^0\ 50'\ 21'',6$, $F = 19380$.

125. $\cos \frac{1}{2}(\beta - \alpha) = h : w$, $\frac{1}{2}(\beta + \alpha) = 90^0 - \frac{1}{2}\gamma$;
$a = 89$, $b = 761$, $c = 840$, $\alpha = 2^0\ 56'\ 15'',4$,
$\beta = 25^0\ 59'\ 21'',2$, $F = 16380$.

126. $\cos\frac{1}{2}(\alpha - \beta) = h : w$, $\operatorname{cotg}\frac{1}{2}\gamma = \operatorname{tg}\frac{1}{2}(\alpha - \beta)(m + n):(m - n)$;
$a = 82$, $b = 30$, $c = 104$, $\alpha = 36^0\ 52'\ 11'',6$,
$\beta = 12^0\ 40'\ 49'',4$, $\gamma = 130^0\ 26'\ 59'',0$, $F = 936$.

127. $c = u + v$; $a^2 = \frac{u}{v}(w^2 + uv)$, $b^2 = \frac{v}{u}(w^2 + uv)$;
$c = 20$, $a = 27{,}276$, $b = 18{,}184$, $\alpha = 91^0\ 3'$,
$\beta = 41^0\ 48'$, $\gamma = 47^0\ 9'$, $F = 181{,}82$.

C.

a) **1.** $b = p\sqrt{\frac{\sin\beta}{\sin\alpha}}$, $a = p\sqrt{\frac{\sin\alpha}{\sin\beta}}$; $a = 102$, $b = 61$,
$c = 109$, $\gamma = 79^0\ 36'\ 40''$, $F = 3060$.

2. $a^2 = \frac{2F\sin\alpha}{\sin\beta\sin\gamma}$, $b^2 = \frac{2F\sin\beta}{\sin\alpha\sin\gamma}$, $c^2 = \frac{2F\sin\gamma}{\sin\alpha\sin\beta}$;
$a = 229$, $b = 221$, $c = 87$, $\gamma = 15^0\ 11'\ 21'',4$.

36. $2r = d : (\sin\gamma - \sin\beta\cos\gamma)$; $a = 370$, $b = 541$,
$c = 421$, $F = 77700$.
(In der Aufgabe ist p_a statt q_a zu lesen.)

37. $a = w\sin(\beta + \frac{1}{2}\gamma) : \sin\beta$, $b = w\sin(\beta + \frac{1}{2}\gamma) : \sin\gamma$;
a) $a = 1{,}08183$, $b = 0{,}99366$, $c = 0{,}71693$,
$F = 0{,}34645$;
b) $a = 869{,}46$, $b = 1003{,}12$, $c = 1156{,}95$,
$F = 423030$.

b) **1.** $\operatorname{tang}\alpha = a : h_a'$; $\alpha = 73^0\ 44'\ 23'',0$, $\gamma = 48^0\ 9'\ 5'',9$,
$a = 424$, $b = 375$, $F = 59220$.

D.

1 — 11 führen zu weitläufigen Entwickelungen und sind für Zahlenbeispiele durch Gleichungen höherer Grade lösbar. Die Ausführung würde hier zu weit führen.

1 — 3 sind unstatthafte Aufgaben, da durch zwei der gegegebenen Stücke das dritte bereits bestimmt, die Aufgabe also entweder sich widersprechend oder doch unbestimmt ist.

§. 28.

1. $x^2 = \frac{a^2 \sin\gamma^2}{\sin(\beta+\gamma)^2} + \frac{a^2 \sin\alpha^2}{\sin(\alpha+\delta)^2} - \frac{2a^2 \sin\gamma \sin\alpha \cos(\delta-\beta)}{\sin(\beta+\gamma)\sin(\alpha+\delta)}$
$= \frac{a^2 \sin\beta^2}{\sin(\beta+\gamma)^2} + \frac{a^2 \sin\delta^2}{\sin(\alpha+\delta)^2} - \frac{2a^2 \sin\beta \sin\delta \cos(\gamma-\alpha)}{\sin(\beta+\gamma)\sin(\alpha+\delta)}$;
a) $x = 2656{,}1$, b) $27{,}75$.

2. $x^2 = \frac{a^2 \sin\delta^2}{\sin(\gamma+\delta)^2} + \frac{a^2 \sin\beta^2}{\sin(\alpha+\beta)^2} - \frac{2a^2 \sin\beta \sin\delta \cos(\alpha+\gamma)}{\sin(\alpha+\beta)\sin(\gamma+\delta)}$
$= \frac{a^2 \sin\gamma^2}{\sin(\gamma+\delta)^2} + \frac{a^2 \sin\alpha^2}{\sin(\alpha+\beta)^2} - \frac{2a^2 \sin\alpha \sin\gamma \cos(\beta+\delta)}{\sin(\alpha+\beta)\sin(\gamma+\delta)}$;
$x = 264{,}1$.

3. a) $a \sin\beta \sin\gamma : \sin(\beta+\gamma) = 554{,}775$;
b) $a \sin\alpha \sin\beta : \sin(\alpha+\beta) - b = 2008{,}72$.

4. $\frac{a \sin\alpha \sin(\gamma-\beta)}{\sin(\alpha+\gamma)\sin(\alpha+\beta)} = 32{,}083$.

5. $\frac{a \sin\alpha \sin(\beta-\gamma)}{\sin(\alpha+\beta)\sin(\alpha+\gamma)} = 4722{,}33$.

6. 4,2258 Ml.

7. $a\sqrt{2} \sin\beta = 5$ M.

8. $x^2 + (a+b)\,x = \frac{ab \sin(\alpha+\beta+\gamma)\sin\beta}{\sin\alpha \sin\gamma}$; $x = 52$.

9. $AB = \frac{a \sin\alpha \sin(\delta-\gamma)}{\sin(\alpha+\delta)\sin(\alpha+\gamma)} = 611$,
$AC = \frac{a \sin\delta \sin(\alpha-\beta)}{\sin(\alpha+\delta)\sin(\beta+\delta)} = 547{,}7$.
$CE = \frac{a \sin\beta}{\sin(\beta+\delta)}$, $BE = \frac{a \sin\alpha}{\sin(\alpha+\gamma)}$,
$BC^2 = CE^2 + BE^2 - 2\,CE.BE.\cos(\delta-\gamma)$, $BC = 244{,}3$.

10. $\sin\varphi = \frac{a \sin\beta}{b}$, $\psi = 180^0 - (\beta+\gamma+\varphi)$,
$AD = x = \frac{b \sin\psi}{\sin(\beta+\gamma-\alpha)} = 830{,}9$,
$CD = y = \frac{b \sin(\alpha+\varphi)}{\sin(\beta+\gamma-\alpha)} = 907{,}9$,
$BD^2 = a^2 + y^2 - 2ay\cos\gamma$, $BD = 782{,}4$.

11. $\cos\beta = (a^2 + c^2 - b^2) : 2ac$,
$x = c \sin(\beta-\delta) : \sin\delta = 59{,}524$.

12. $W^2 = a^2 \sin\beta^2 + b^2 \sin\alpha^2 - 2ab \sin\alpha \sin\beta \cos(\alpha+\beta)$,
$AD = a(a+b)\sin\beta : W = 229$,
$BD = ab \sin(\alpha+\beta) : W = 61$,
$CD = b(a+b)\sin\alpha : W = 109$.

13. Berechnet man aus den Seiten des Dreiecks ABC die Winkel $ABC = \beta$, $BAC = \alpha$, setzt $AB = c$, $AC = b$, $BC = a$, $BDA = \varphi$, $ADC = \psi$, ferner $\operatorname{tg} \vartheta = \sin\gamma . \sin(\psi - \beta) : \sin\beta . \sin(\varphi - \gamma)$, $\operatorname{tg} \frac{1}{2}(\xi - \eta) = \operatorname{cotg} \frac{1}{2}(\varphi + \psi) \operatorname{cotg}(45^0 + \vartheta)$, $\xi + \eta = 180^0 - (\varphi + \psi)$, so ist $x = a \sin\eta : \sin(\varphi + \psi) = 629$, $y = c \sin(\beta + \xi) : \sin\varphi = 680$.

14. $AP = \frac{a \sin\delta}{\sin(\alpha + \beta + \gamma + \delta)}$, $BP = \frac{a \sin(\delta + \varepsilon)}{\sin(\alpha + \beta + \delta + \varepsilon)}$,
$BQ = \frac{a \sin(\alpha + \beta)}{\sin(\alpha + \beta + \delta + \varepsilon)}$, $CQ = \frac{a \sin\alpha}{\sin(\alpha + \delta + \varepsilon + \zeta)}$,
$PC = \frac{a \sin(\delta + \varepsilon + \zeta)}{\sin(\alpha + \delta + \varepsilon + \zeta)}$,
$AB^2 = AP^2 + BP^2 - 2AP . BP . \cos\gamma$,
$BC^2 = BQ^2 + CQ^2 - 2BQ . CQ \cos\zeta$,
$AC^2 = AP^2 + PC^2 - 2AP . PC . \cos(\beta + \gamma)$;
$AB = 19211$, $BC = 9207$, $AC = 27366$.

15. $CD^2 = \frac{c}{a}(ac - b^2)$, $BD^2 = \frac{a}{c}(ac - b^2)$,
$\cos\alpha = \frac{b(a + c)}{2ac}$ für $AB = a$, $AD = b$, $AC = c$;
$CD = 20$, $BD = 10$, $\alpha = 30^0$.

16. $x^2 = b^2 - 4a \operatorname{tang} \frac{1}{2}\alpha$, $x = 428{,}845$.

17. $AC^2 = a^2 + c^2 - 2ac \cos\beta$, $AC = 0{,}8482$;
$\sin BAC = a \sin\beta : AC$, $BAC = 67^0\, 21'$;
$\cos DAC = (d^2 + AC^2 - e^2) : 2d . AC$,
$DAC = 21^0 4' 12'',5$; $BC^2 = c^2 + d^2 - 2cd \cos(BAC + DAC)$,
$BD = 0{,}85704$; $\sin ADB = c \sin(BAC + DAC) : BD$,
$ADB = 71^0 50' 55''$; $\sin ABD = d \sin(BAC + DAC) : BD$,
$ABD = 19^0\, 33'\, 52''$; $x = BD - f - g = 0{,}654$.

18. Setze $W^2 = \sin\gamma^2 \sin(\beta + \gamma + \delta)^2 + \sin(\gamma + \delta)^2 . \sin(\alpha + \beta + \gamma)^2 - 2 \sin\gamma \sin(\gamma + \delta) \sin(\alpha + \beta + \gamma) . \sin(\beta + \gamma + \delta) \cos\alpha$, so ist
$x = a \sin(\alpha + \beta + \gamma) \sin(\beta + \gamma + \delta) : W = 229$.

19. $\frac{\sqrt{4(a^2 + b^2 - 2ab\cos\gamma) - (a + b)^2 \sin\gamma} - 2(a - b) \cos \frac{1}{2}\gamma^2}{4 \sin \frac{1}{2}\gamma}$.

20. $a \sin\alpha \sin\beta : \sin(\beta - \alpha)$; a) 295,5; b) 525,89; c) 196,97.

21. $a \sin\alpha \sin\beta : \sin(\alpha - \beta) = 368{,}08$.

22. $AE = b \sin\beta : \sin(\alpha - \beta)$, $AD = b \sin\alpha : \sin(\alpha - \beta)$, $x^2 = a^2 + AE^2 - 2a \,.\, AE \,.\cos\alpha = (a + b)^2 + AD^2 - 2(a + b) \,.\, AD \,.\cos\beta$, $x = 762{,}03$.

23. $\frac{a}{2\sin(\beta - \alpha)} \sqrt{\sin\alpha^2 + \sin(\beta - \alpha)^2 - 2\sin\alpha \sin(\beta - \alpha)\cos\beta}$ $= 457$.

24. Höhe: $h \sin\alpha : \cos\beta \sin(\alpha + \beta) = 151{,}01$,
Entf.: $h \operatorname{tang}\beta = 299{,}59$.

25. $a \sin\beta \operatorname{tang}\gamma : \sin(\alpha + \beta) = a \sin\alpha \operatorname{tang}\delta : \sin(\alpha + \beta) = 113{,}97$; $\sin\beta \operatorname{tang}\gamma = \sin\alpha \operatorname{tang}\delta$.

26. $AB = a$, $AH = b$, $BH = c$,

$$x^2 = \frac{a^2 - (b^2 + c^2)\sin\alpha^2 + \cos\alpha\sqrt{a^4 - (b^2 - c^2)^2 \sin\alpha^2}}{2\sin\alpha^2},$$

$x = 132{,}3$.

27. $\operatorname{cotg} x = \sqrt{a^2 \operatorname{cotg}\beta^2 + b^2 - ab \operatorname{cotg}\beta\sqrt{2}} : a$, $x = 42^0\ 0'\ 5''$.

28. $a : \sqrt{\operatorname{cotg}\alpha^2 + \operatorname{cotg}\beta^2 - 2\operatorname{cotg}\alpha \operatorname{cotg}\beta \cos 22^0 30'} = a \sin\alpha \sin\beta : \sqrt{\sin(\alpha + \beta)^2 - \sin 2\alpha \,.\, \sin 2\beta \,.\, \cos\tfrac{1}{2}\varphi^2} = 18{,}503$.

29. Höhe: $x = c \sin\alpha \sin\beta :$
$\sqrt{\sin\alpha^2 + \sin\beta^2 - 2\sin\alpha \,.\, \sin\beta \,.\, \cos\gamma} = 1584{,}56$,
Entf. von A, $x : \sin\alpha = 1661{,}46$: von B,
$x : \sin\beta = 3145{,}36$.

30. $$x^2 = \frac{-ab\sin\beta^2 \,.\, \sin\alpha^2 \,.\, \sin\gamma^2 \,.\, (a + b)}{b\sin(\alpha + \beta)\sin(\alpha - \beta)\sin\gamma^2 + a\sin(\beta + \gamma)\sin(\gamma - \beta)\sin\alpha^2},$$
$x = 10$.

31. $a \sin(\beta + \delta)\sin(\gamma + \delta) : \sin(\gamma - \beta)\cos\delta = 60$.

32. $\frac{a\sin\alpha\sin(\beta - \varphi)}{\sin(\alpha + \beta)} + b = \frac{a\sin\beta\sin(\alpha + \varphi)}{\sin(\alpha + \beta)} = 264$,
$\sin\varphi = \frac{b}{a}$.

33. $\sin\varphi = \frac{c - b}{a}$, $x = \frac{a\sin(\alpha - \varphi)\sin\beta}{\sin(\beta - \alpha)} + c$
$= \frac{a\sin(\beta - \varphi)\sin\alpha}{\sin(\beta - \alpha)} + b = 99$.

34. $12^m{,}052$.

35. $c^2 = a^2 \operatorname{cotg}\alpha^2 + b^2 \operatorname{cotg}\beta^2 - 2ab \operatorname{cotg}\alpha \operatorname{cotg}\beta \cos\gamma$,
$x^2 = (a - b)^2 + c^2$; $x = 203{,}06$.

36. $h\sqrt{\operatorname{tang}\alpha^2 + \operatorname{tang}\beta^2 - 2\operatorname{tang}\alpha \operatorname{tang}\beta\cos\gamma} = 276{,}14$.

13. Berechnet man aus den Seiten des Dreiecks ABC die Winkel $ABC = \beta$, $BAC = \alpha$, setzt $AB = c$, $AC = b$, $BC = a$, $BDA = \varphi$, $ADC = \psi$, ferner $\operatorname{tg} \vartheta = \sin\gamma . \sin(\psi - \beta) : \sin\beta . \sin(\varphi - \gamma)$, $\operatorname{tg} \frac{1}{2}(\xi - \eta) = \operatorname{cotg} \frac{1}{2}(\varphi + \psi) \operatorname{cotg}(45^0 + \vartheta)$, $\xi + \eta = 180^0 - (\varphi + \psi)$, so ist $x = a \sin\eta : \sin(\varphi + \psi) = 629$, $y = c \sin(\beta + \xi) : \sin\varphi = 680$.

14. $AP = \frac{a \sin\delta}{\sin(\alpha + \beta + \gamma + \delta)}$, $BP = \frac{a \sin(\delta + \varepsilon)}{\sin(\alpha + \beta + \delta + \varepsilon)}$,
$BQ = \frac{a \sin(\alpha + \beta)}{\sin(\alpha + \beta + \delta + \varepsilon)}$, $CQ = \frac{a \sin\alpha}{\sin(\alpha + \delta + \varepsilon + \zeta)}$,
$PC = \frac{a \sin(\delta + \varepsilon + \zeta)}{\sin(\alpha + \delta + \varepsilon + \zeta)}$,
$AB^2 = AP^2 + BP^2 - 2AP . BP . \cos\gamma$,
$BC^2 = BQ^2 + CQ^2 - 2BQ . CQ \cos\zeta$,
$AC^2 = AP^2 + PC^2 - 2AP . PC . \cos(\beta + \gamma)$;
$AB = 19211$, $BC = 9207$, $AC = 27366$.

15. $CD^2 = \frac{c}{a}(ac - b^2)$, $BD^2 = \frac{a}{c}(ac - b^2)$,
$\cos\alpha = \frac{b(a + c)}{2ac}$ für $AB = a$, $AD = b$, $AC = c$;
$CD = 20$, $BD = 10$, $\alpha = 30^0$.

16. $x^2 = b^2 - 4a \operatorname{tang} \frac{1}{2}\alpha$, $x = 428{,}845$.

17. $AC^2 = a^2 + c^2 - 2ac \cos\beta$, $AC = 0{,}8482$;
$\sin BAC = a \sin\beta : AC$, $BAC = 67^0\,21'$;
$\cos DAC = (d^2 + AC^2 - e^2) : 2d . AC$,
$DAC = 21^0 4' 12'',5$; $BC^2 = c^2 + d^2 - 2cd \cos(BAC + DAC)$,
$BD = 0{,}85704$; $\sin ADB = c \sin(BAC + DAC) : BD$,
$ADB = 71^0 50' 55''$; $\sin ABD = d \sin(BAC + DAC) : BD$,
$ABD = 19^0\,33'\,52''$; $x = BD - f - g = 0{,}654$.

18. Setze $W^2 = \sin\gamma^2 \sin(\beta + \gamma + \delta)^2 + \sin(\gamma + \delta)^2 . \sin(\alpha + \beta + \gamma)^2 - 2 \sin\gamma \sin(\gamma + \delta) \sin(\alpha + \beta + \gamma) . \sin(\beta + \gamma + \delta) \cos\alpha$, so ist
$x = a \sin(\alpha + \beta + \gamma) \sin(\beta + \gamma + \delta) : W = 229$.

19. $\frac{\sqrt{4(a^2 + b^2 - 2ab\cos\gamma) - (a + b)^2 \sin\gamma} - 2(a - b)\cos\frac{1}{2}\gamma^2}{4 \sin\frac{1}{2}\gamma}$.

20. $a \sin\alpha \sin\beta : \sin(\beta - \alpha)$; a) 295,5; b) 525,89; c) 196,97.

21. $a \sin\alpha \sin\beta : \sin(\alpha - \beta) = 368{,}08$.

22. $AE = b \sin\beta : \sin(\alpha-\beta)$, $AD = b \sin\alpha : \sin(\alpha-\beta)$, $x^2 = a^2 + AE^2 - 2a\,.\,AE\,.\cos\alpha = (a+b)^2 + AD^2 - 2(a+b)\,.\,AD\,.\cos\beta$, $x = 762{,}03$.

23. $\frac{a}{2\sin(\beta-\alpha)}\sqrt{\sin\alpha^2 + \sin(\beta-\alpha)^2 - 2\sin\alpha\sin(\beta-\alpha)\cos\beta} = 457$.

24. Höhe: $h\sin\alpha : \cos\beta\sin(\alpha+\beta) = 151{,}01$,
Entf.: $h \operatorname{tang}\beta = 299{,}59$.

25. $a\sin\beta\operatorname{tang}\gamma : \sin(\alpha+\beta) = a\sin\alpha\operatorname{tang}\delta : \sin(\alpha+\beta) = 113{,}97$; $\sin\beta\operatorname{tang}\gamma = \sin\alpha\operatorname{tang}\delta$.

26. $AB = a$, $AH = b$, $BH = c$,

$$x^2 = \frac{a^2 - (b^2+c^2)\sin\alpha^2 + \cos\alpha\sqrt{a^4 - (b^2-c^2)^2\sin\alpha^2}}{2\sin\alpha^2},$$

$x = 132{,}3$.

27. $\operatorname{cotg} x = \sqrt{a^2\operatorname{cotg}\beta^2 + b^2 - ab\operatorname{cotg}\beta\sqrt{2}} : a$, $x = 42^0\,0'\,5''$.

28. $a : \sqrt{\operatorname{cotg}\alpha^2 + \operatorname{cotg}\beta^2 - 2\operatorname{cotg}\alpha\operatorname{cotg}\beta\cos 22^0 30'} = a\sin\alpha\sin\beta : \sqrt{\sin(\alpha+\beta)^2 - \sin 2\alpha\,.\,\sin 2\beta\,.\cos\tfrac{1}{2}\varphi^2} = 18{,}503$.

29. Höhe: $x = c\sin\alpha\sin\beta : \sqrt{\sin\alpha^2 + \sin\beta^2 - 2\sin\alpha\,.\,\sin\beta\,.\cos\gamma} = 1584{,}56$,
Entf. von A, $x : \sin\alpha = 1661{,}46$: von B,
$x : \sin\beta = 3145{,}36$.

30. $$x^2 = \frac{-ab\sin\beta^2\,.\sin\alpha^2\,.\sin\gamma^2\,.\,(a+b)}{b\sin(\alpha+\beta)\sin(\alpha-\beta)\sin\gamma^2 + a\sin(\beta+\gamma)\sin(\gamma-\beta)\sin\alpha^2},$$
$x = 10$.

31. $a\sin(\beta+\delta)\sin(\gamma+\delta) : \sin(\gamma-\beta)\cos\delta = 60$.

32. $\frac{a\sin\alpha\sin(\beta-\varphi)}{\sin(\alpha+\beta)} + b = \frac{a\sin\beta\sin(\alpha+\varphi)}{\sin(\alpha+\beta)} = 264$,
$\sin\varphi = \frac{b}{a}$.

33. $\sin\varphi = \frac{c-b}{a}$, $x = \frac{a\sin(\alpha-\varphi)\sin\beta}{\sin(\beta-\alpha)} + c = \frac{a\sin(\beta-\varphi)\sin\alpha}{\sin(\beta-\alpha)} + b = 99$.

34. $12^m{,}052$.

35. $c^2 = a^2\operatorname{cotg}\alpha^2 + b^2\operatorname{cotg}\beta^2 - 2ab\operatorname{cotg}\alpha\operatorname{cotg}\beta\cos\gamma$, $x^2 = (a-b)^2 + c^2$; $x = 203{,}06$.

36. $h\sqrt{\operatorname{tang}\alpha^2 + \operatorname{tang}\beta^2 - 2\operatorname{tang}\alpha\operatorname{tang}\beta\cos\gamma} = 276{,}14$.

37. $\frac{h}{\sin\beta\sin\gamma}\sqrt{\sin\beta^2+\sin\gamma^2-2\sin\beta\sin\gamma\cos\alpha}=383{,}05$.

38. $BC=\frac{a\sin\beta}{\sin(\alpha+\beta)}=205$; $AC=\frac{a\sin\alpha\sin\beta}{\sin(\alpha+\beta)\sin(\alpha+\gamma)}=445$;
$AB=\frac{a\sin\beta\sin\gamma}{\sin(\alpha+\beta)\sin(\alpha+\gamma)}=624$,
Höhe von B gleich $AB.\sin\delta=38{,}925\,(39)$,
von C gleich $BC.\sin\varepsilon+AB.\sin\delta=84$.

39. $R=133{,}54$, $\sphericalangle P\,|\,R=41^0\,55'\,32'',2$,
$\sphericalangle Q\,|\,R=30^0\,4'\,27'',8$.

40. $P\,|\,R=56^0$, $Q\,|\,R=57^0\,20'$.

41. $P=5169{,}9$, $Q=9632{,}5$.

42. $Q=244$, $Q\,|\,R=26^0\,47'\,5'',3$.

43. $Q=137$, $P\,|\,R=26^0\,47'\,7'',6$.

44. $R=3700$, $Q=1201$.

45. $x=\frac{(a+2b)(a-b)}{6(a+b)\sin(\alpha+\beta)}\sqrt{\sin\alpha^2+\sin\beta^2-2\sin\alpha\sin\beta\cos(\alpha+\beta)}$;
a) $\frac{(a+2b)(a-b)}{6(a+b)}\tang\alpha$; b) $\frac{a}{6}\;(=\frac{1}{2}c)$;
c) $\frac{a}{6\sin(\alpha+\beta)}\sqrt{\sin\alpha^2+\text{etc., wie vorher}}$.

46. $c^2=a^2+b^2-2ab\cos\gamma$, $c=123$;
$d=b(b-a\cos\gamma):2c=16{,}5$; $x=\frac{2}{3}(c-d)=71$,
$y=\frac{1}{3}(c+2d)=52$.

49. $gt^2\sin(45^0-\frac{1}{2}\alpha)^2=36{,}586$.

50. $333\sqrt{2b^2+2bc+c^2-2b(b+c)\cos\alpha}=4550{,}9$.

51. $\sin\beta=a\sin\alpha:l$, $\varphi=180^0-(\alpha+\beta)=79^0\,20'$.

52. a) $90^0-\alpha$, b) $180^0-\alpha$, c) $180^0-2\alpha$.

53. a) $AC=\frac{2r\sin\frac{1}{2}\beta\cos(\delta\pm\frac{1}{2}\beta)}{\sin(\gamma-\beta\mp\delta)}$,
$x^2=AC^2+r^2+2r\,AC.\cos\gamma$; b) $59{,}336$.

54. $12\frac{12}{29}=12{,}4138$.

55. $\tang\alpha=\frac{3b+2x}{5a-2y}$, oder $\frac{3b+2x}{3a+2y}$; $20^0\,11'\,40''$ oder
$29^0\,18'\,39'',2$. **56.** $\sqrt{a^2+ab+b^2}$.

57. Es ist gleichseitig. **58.** $F_1=10{,}3445$.

59. $n=3$ oder 1; $F=\frac{1}{6}hh':\sin\gamma$ oder $\frac{1}{2}hh':\sin\gamma$.

60. Ist F der Flächeninhalt des gegebenen Dreiecks, so ist
$S = \sqrt{\tfrac{1}{2}(a^2 + b^2 + c^2) + 6F : \sqrt{3}}$,
$\Sigma = \tfrac{1}{2}(a^2 + b^2 + c^2) - 2F : \sqrt{3}$.

61. $\alpha = 180^0 - \alpha_1$; $\beta = \alpha_1 + \gamma_1 - 180^0$; $\gamma = 180^0 - \gamma_1$;
$a = \frac{b_1 - a_1 \cos\gamma_1}{\sin\gamma_1} + \frac{c_1 - a_1\cos\beta_1}{\sin\beta_1}$, etc.
$a = 62{,}481$; $b = 52{,}090$; $c = 52{,}045$.

62. $\operatorname{tang}\varphi = \frac{m+n}{m-n}\operatorname{tang}\alpha$, $\varphi = 45^0$.

63. $\cos(2x_1 + \gamma) = \sqrt{2}\,.\,\sin(45^0 - \gamma)$;
$\cos(2x_{11} - \gamma) = \sqrt{2}\,.\,\cos(45^0 - \gamma)$; $x_1 = x_{11} = 45^0$.

64. $a^2(\sin\alpha - \tfrac{1}{4}\pi)$; $a^2(1 - \tfrac{1}{4}\pi) = 0{,}214602\, a^2$.

65. $\tfrac{1}{2}b \pm \sqrt{4b^2 - a(a-b)\operatorname{tang}\tfrac{1}{2}\beta^2}$; 1,2; 0,05.

66. $R = c\sin\tfrac{1}{2}(\beta + \alpha)\cos\tfrac{1}{2}(\beta - \alpha) = 392{,}42$;
$r = c\cos\tfrac{1}{2}(\beta + \alpha)\,.\,\sin\tfrac{1}{2}(\beta - \alpha) = 170{,}99$.

67. $\cos\alpha = \frac{d^2 + (\varrho + r)^2 - (\varrho + R)^2}{2d(\varrho + r)}$, $\cos\beta = \frac{(\varrho + r)^2 + (\varrho + R)^2 - d^2}{2(\varrho + r)(\varrho + R)}$,
$\alpha + \tfrac{1}{2}\beta - 90^0 = 29^0\ 49'\ 44''$.

68. $\pm\frac{d^2 + R^2 - r^2 - 2dR\cos\alpha}{2(r \mp R \pm d\cos\alpha)}$.

69. $x^2 = 2a^2\operatorname{tang}\alpha\sin\beta\,\frac{\cos(\beta - \alpha) \pm \sqrt{\sin\beta\sin(2\alpha - \beta)}}{\cos\alpha}$.

70. Setzt man $\frac{r}{a} = p$, $\frac{\sin\alpha \pm p\cos(\alpha - \gamma)}{\sin(\alpha - \gamma)} = q$, so ist
$q\sin\varphi \pm p\cos\varphi = 1$, und für $\frac{p}{q} = \operatorname{tang}\gamma$,
$\sin(\varphi \pm \gamma) = \frac{\cos\beta}{q}$.

71. Setzt man $AC = a$, $BC = b$, $\sphericalangle ACB = \gamma$, $\sphericalangle ACO = \varphi$,
so ist für $\frac{a(b^2 - r^2)}{b(a^2 - r^2)\sin\gamma} - \operatorname{cotg}\gamma = \operatorname{cotg}\vartheta$,
$\cos(\vartheta + \varphi) = \frac{r(b^2 - a^2)\sin\vartheta}{b(a^2 - r^2)\sin\gamma}$.

72. Er ist um $0^0\ 6'\ 38''$ zu klein.

73. Er ist um $c\,.\,0^0\ 1'\ 3''$ zu klein.

74. Um $1^0\ 42'\ 5'',8$ zu gross. **75.** Um $0^0\ 2'\ 30''$ zu klein.

76. Um $0^0\ 5'\ 25''$ zu gross.

77. Um $0^0\ 12'\ 28''$. Um $0{,}0072\,a$.

§. 29.

a) 1. $f = 99$, $F = 4116$, $\alpha = 123^0 8' 48'',4$, $\beta = 59^0 29' 23'',1$, $\gamma = 145^0 57' 32'',2$, $\delta = 31^0 24' 16'',3$.

2. $c = 12{,}0415$, $d = 10{,}8168$, $f = 16$, $F = 112$, $\alpha = 105^0 42' 32'',2$, $\beta = 89^0 24' 53'',0$, $\gamma = 89^0 33' 8'',3$, $\alpha_2 = 49^0 23' 55'',4$, $\beta_1 = 40^0 36' 4'',7$, $\beta_2 = 48^0 48' 50'',4$, $\gamma_1 = 41^0 11' 9'',5$, $\delta_1 = 41^0 38' 2'',1$, $\delta_2 = 33^0 41' 24'',4$.

3. $b = 12{,}0415$, $d = 12{,}2066$, $f = 18$, $F = 144$, $\alpha = 103^0 49' 19'',4$, $\beta = 89^0 33' 7'',4$, $\gamma = 89^0 38' 48'',1$, $\delta = 76^0 58' 45'',1$, ferner die Winkel α_1, β_1, β_2, γ_1, δ_1, δ_2 der Reihe nach: $55^0\ 0'\ 28'',9$; $41^0\ 11'\ 9'',5$; $48^0\ 21'\ 57'',9$; $41^0 38' 2'',1$; $41^0 59' 14'',0$; $34^0\ 59' 31'',1$.

4. Für das eine der beiden Dreiecke ist: $d = 7{,}07109$, $e = 6$, $f = 10$, $F = 30$, $\alpha = 153^0\ 26'\ 9'',1$, $\gamma = 85^0\ 25'\ 34'',0$, α_1 bis $\delta_2 = 81^0\ 52'\ 11'',8$; $71^0\ 33'\ 57'',3$; $18^0\ 26'\ 2'',7$; $59^0\ 2'\ 10'',4$; $30^0\ 57'\ 49'',6$, $54^0\ 27'\ 44'',4$; $35^0\ 32'\ 15'',6$; $8^0\ 7'\ 48'',2$.

5. $c = 5$, $d = 4{,}12318$, $e = 4$, $f = 6$, $F = 12$, $\alpha = 139^0 23' 55'',5$, $\gamma = 86^0 49' 12'',8$, $\delta = 50^0 54' 21'',8$; α_2 bis δ_2 $63^0\ 26'\ 5'',7$; $26^0\ 33'\ 54'',3$; $56^0\ 18'\ 35'',6$; $33^0\ 41'\ 24'',4$; $36^0\ 52'\ 11'',6$; $14^0\ 2'\ 10'',2$.

6. Zweideutig, das Zahlenbeispiel eindeutig; $b = 5{,}831$, $c = 7{,}81033$, $d = 6{,}08271$, $f = 9$, $F = 27$, $\alpha = 152^0\ 6'\ 12'',7$; α_1 u. s. w.: $80^0\ 32'\ 15'',4$; $71^0\ 33'\ 57'',3$; $18^0\ 26'\ 2''7$; $59^0\ 2'\ 10'',4$; $30^0\ 57'\ 49'',6$; $50^0\ 11'\ 39'',5$; $39^0\ 48'\ 20'',5$; $9^0\ 27'\ 44'',6$.

7. $d = 24$, $F = 132$, $\alpha = 36^0\ 52'\ 11'',6$, $\delta = 61^0 55' 39'',1$.

8. Zweideutig, das Beispiel eindeutig: $c = 11{,}4018$, $e = 10$, $f = 14$, $F = 70$, $\alpha = 130^0\ 36'\ 7'',7$, $\beta = 85^0 25' 34'',0$, $\gamma = 87^0 39' 45'',8$, $\delta = 56^0 18' 32'',5$, α_1 etc.: $71^0 33' 57'',3$; $54^0\ 27'\ 44'',4$; $35^0\ 32'\ 15'',6$; $52^0 7' 30'',2$; $37^0 52' 29'',8$; $18^0\ 26'\ 2'',7$.

9. $d_1 = 10{,}7703$, $e = 12$, $f = 17$, $F = 102$, $\alpha = 128^0\ 27' 14'',0$, $\gamma = 92^0 31' 35'',1$, $\delta = 60^0 27' 39'',2$; α_1 etc.: $68^0\ 11'\ 52'',2$; $60^0\ 15'\ 18'',75$; $29^0\ 44'\ 41'',25$; $48^0\ 48'\ 50'',5$; $41^0\ 11'\ 9'',5$; $51^0\ 20'\ 25'',6$; $21^0\ 48'\ 4'',8$.

10. $b = 7{,}211$, $d = 9{,}2196$, $e = 8$, $f = 13$, $F = 52$, $\beta = 82^0\ 52'\ 29'',9$, $\gamma = 90^0$, α_2 etc.: $63^0\ 26'\ 5'',7$; $26^0\ 33'\ 54'',3$; $56^0\ 18'\ 35'',6$; $33^0\ 41'\ 24'',4$; $56^0\ 18'\ 35'',6$; $33^0\ 41'\ 24'',4$; $12^0\ 31'\ 44'',0$.

11. $a_1 = 2{,}2361$, $b = 4{,}47215$, $c = 6{,}40314$, $\alpha = 142^0\ 7'\ 30'',3$, $\beta = 90^0$, $\gamma = 78^0\ 54'\ 19'',9$, $\delta = 49^0\ 49'\ 9'',9$; $F = 17{,}5$; α_1 etc.: $78^0\ 41'\ 24'',6$; $63^0\ 26'\ 5'',7$; $26^0\ 33'\ 54'',3$; $63^0\ 26'\ 5'',7$; $51^0\ 20'\ 25'',6$; $38^0\ 39'\ 34'',4$.

12. $c = 10{,}8168$, $d = 9{,}0556$, $e = 7$, $f = 12$, $F = 42$, $\alpha = 155^0\ 13'\ 32'',3$, $\gamma = 82^0\ 52'\ 29'',9$, $\delta = 40^0\ 1'\ 49'',4$; α_2 etc. : $71^0\ 33'\ 57'',3$; $18^0\ 26'\ 2'',7$; $63^0\ 26'\ 5'',7$; $26^0\ 33'\ 54'',3$; $56^0\ 18'\ 35'',6$; $33^0\ 41'\ 24'',4$.

b) 1. Aus $\triangle\ ABC$ bestimme e, α_2, γ_1, setze $r = a : 2 \sin \delta_2$, $p^2 = e^2 + r^2 - 2er \sin(\alpha_2 + \delta_2)$, $\sin \vartheta = r \cos(\alpha_2 + \delta_2) : p$, $\cos \eta = (c^2 + p^2 - r^2) : 2cp$, so ist $\gamma_2 = \vartheta \pm \eta$; berechne dann das $\triangle\ ACD$, u. s. w. $d = 7{,}07109$, $e = 6$, $f = 9$, $F = 27$, $\alpha = 145^0\ 18'\ 17'',5$, $\gamma = 76^0\ 15'\ 49'',2$, $\delta = 43^0\ 40'\ 3'',8$; α_1 etc. : $81^0\ 52'\ 11'',8$; $63^0\ 26'\ 5'',7$; $26^0\ 33'\ 54'',3$; $68^0\ 11'\ 55'',2$; $21^0\ 48'\ 4'',8$; $54^0\ 27'\ 44'',4$; $35^0\ 32'\ 15'',6$.

2. Aus $\triangle\ ABC$ ergeben sich e, α_2, γ_1. Setzt man $\alpha_2 + \beta_1 - \delta = \sigma$, so ist $e \cos(\sigma + 2\delta_2) = e \cos \sigma - 2a \sin \beta_1 \sin \delta = e \cos(\sigma - 2\gamma_2)$, u. s. w. $c = 9{,}2196$, $d = 7{,}07109$, $e = 7$, $f = 9$, $\alpha = 145^0\ 18'\ 17'',5$, $\gamma = 67^0\ 50'\ 1'',1$, $F = 31{,}5$; α_1 etc.: $81^0\ 52'\ 11'',8$; $63^0\ 26'\ 5'',7$; $71^0\ 33'\ 54'',3$; $18^0\ 26'\ 5'',7$; $49^0\ 23'\ 55'',4$; $40^0\ 36'\ 4'',7$; $8^0\ 7'\ 48'',2$.

3. $\sin \alpha : \sin \gamma = b \sin \delta_2 : a \sin \delta_1 = \operatorname{tg} \varphi$; $\operatorname{tang} \frac{1}{2}(\alpha - \gamma) = \operatorname{tang} \frac{1}{2}(\alpha + \gamma) \operatorname{tang}(\varphi - 45^0)$, $\frac{1}{2}(\alpha + \gamma) = 180^0 - \frac{1}{2}(\delta_1 + \delta_2 + \beta)$; $c = 9{,}434$, $d = 8{,}06225$, $e = 6$, $f = 11$, $F = 33$, $\alpha = 154^0\ 26'\ 27'',3$, $\gamma = 88^0\ 57'\ 30'',0$, $\delta = 39^0\ 7'\ 49'',6$ α_1 etc. : $82^0\ 52'\ 30'',0$; $71^0\ 33'\ 57'',3$; $18^0\ 26'\ 2'',7$; $59^0\ 2'\ 10'',4$; $30^0\ 57'\ 49'',6$; $57^0\ 59'\ 40'',4$.

c) 1. $\delta = 360^0 - (\alpha + \beta + \gamma)$; $b = \frac{c \sin \delta - a \sin \alpha}{\sin(\gamma + \delta)}$,

$d = \frac{c \sin\gamma - a \sin\beta}{\sin(\gamma + \delta)}$, $F = \frac{c^2 \sin\delta \sin\gamma - a^2 \sin\alpha \sin\beta}{2 \sin(\gamma + \delta)^2}$;
$b = 29$, $d = 84$, $F = 614$, $\delta = 12^0\, 40'\, 49'',4$.

2. $\sin\varphi = (a \sin\alpha - c \sin\delta) : b$, $\beta = 180^0 - (\alpha + \varphi)$,
$\gamma = \varphi + 180^0 - \delta$,
$d = \frac{b \sin(\alpha+\varphi) + c \sin(\alpha+\delta)}{\sin\alpha} = \frac{a \sin(\alpha+\delta) + b \sin(\delta - \varphi)}{\sin\delta}$,
$d = 9,4868$, $e = 7$, $f = 13$, $F = 45,5$, $\beta = 70^0\, 20'\, 45'',8$,
$\gamma = 90^0$.

3. $\delta = 360^0 - (\alpha + \beta + \gamma)$, $\delta_2 = \delta - \delta_1$,
$\beta_2 = 180^0 - (\gamma + \delta_1)$, $\beta_1 = \beta - \beta_2$, $v = \sin\alpha \sin\gamma :$
$\sqrt{\sin\alpha^2 \sin\delta_1{}^2 + \sin\gamma^2 \sin\delta_2{}^2 - 2 \sin\alpha \sin\gamma \sin\delta_1 \sin\delta_2 \cos\beta}$,
$a = \frac{v \,.\, \sin\delta_2}{\sin\alpha} \cdot e$, $b = \frac{v \,.\, \sin\delta_1}{\sin\gamma} \cdot e$, $c = \frac{v \,.\, \sin\beta_2}{\sin\gamma} \cdot e$,
$d = \frac{v \,.\, \sin\beta_1}{\sin\alpha} \cdot e$; $a = 15,81145$, $b = 29,155$, $c = 51,478$,
$d = 47,434$, $f = 60$, $\delta = 35^0\, 23'\, 41'',7$, $F = 900$.

4. $a = 3,6055$, $b = 5,831$, $d = 6,32457$, $e = 7$, $f = 9$,
$F = 31,5$, $\alpha = 127^0\, 52'\, 32'',9$, $\gamma = 102^0\, 31'\, 46'',9$,
$\delta = 48^0\, 14'\, 23'',2$.

5. $a = 5,3851$, $b = 8,6023$, $c = 11,4018$, $d = 9,2196$,
$e = 9$, $\alpha = 145^0\, 40'\, 11'',2$, $\gamma = 87^0\, 39'\, 45'',76$,
$\delta = 50^0\, 24'\, 13'',8$, $F = 63$.

6. Es ist $\sin\alpha_1 \,.\, \sin\beta_1 \,.\, \sin\gamma_1 \,.\, \sin\delta_1 = \sin\alpha_2 \,.\, \sin\beta_2 \,.\, \sin\gamma_2$
$.\, \sin\delta_2 = P$, $\alpha_2 + \beta_2 = 180^0 - (\beta_1 + \gamma_1)$,
$(\gamma_2 + \delta_2) = 180^0 - (\alpha_1 + \delta_1)$, daher für $\alpha_2 - \beta_2 = x$, $\gamma_2 - \delta_2 = y$,
$4P = [\cos x + \cos(\beta_1 + \gamma_1)] \cdot [\cos y - \cos(\alpha_1 + \delta_1)]$
und $y - x = \alpha_1 + \beta_1 - \gamma_1 - \delta_1$, u. s. w.

7. $\cos(\alpha_1 - \gamma + 2\gamma_2) = \frac{2a \sin\beta \sin\alpha_1}{c} + \cos(\alpha_1 + \gamma)$.

8. $\alpha_1 = \beta_2 + \gamma_1 - \delta_2$, $\gamma_1 + \alpha_2 = x$,
$A \cos 2x + B \sin 2x = C$ für
$A = a^2 (\sin\delta_2{}^2 - 2 \cos\alpha_1 \cos\beta_2 \sin\gamma_1 \sin\delta_2 + \sin\gamma_1{}^2)$,
$B = 2a^2 \sin\gamma_1 \sin\beta_2 (\cos\alpha_1 \sin\delta_2 - \sin\gamma_1 \cos\beta_2)$,
$C = A - c^2 \sin\gamma_1{}^2 \sin\delta_2{}^2$.

9. $\delta = 180^0 - (\alpha_1 + \gamma_2)$, $\delta_2 = \delta - \delta_1$,
$\sin\alpha_1 \sqrt{a^2 - f^2 \sin\delta_2{}^2} - \sin\gamma_2 \sqrt{b^2 - f^2 \sin\delta_1{}^2}$
$= f(\sin\gamma_2 \cos\delta_1 - \sin\alpha_1 \cos\delta_2)$, u. s. w.

B a) 1. $\sin\alpha = F : ab$. $c_1 = 867$, $\alpha = 95^0\,22'\,18'',4$,
$\alpha_2 = 33^0\,51'\,18'',1$, $\alpha_1 = 61^0\,22'\,0'',3$, $\beta_1 = 30^0\,52'\,11'',4$,
$\beta_2 = 53^0\,55'\,30'',2$, $\varphi = 64^0\,43'\,29'',5$, $f = 941{,}357$.

2. $b = F : a\sin\alpha = 509$, $e = 555$, $f = 881{,}014$,
$\alpha_2 = 55^0\,47'\,40'',4$, $\alpha_1 = 59^0\,48'\,50'',3$, $\beta_1 = 31^0\,23'\,56'',6$,
$\beta_2 = 32^0\,59'\,32'',7$, $\varphi = 87^0\,11'\,37'',0$.

3. $\sin\varphi = 2F : ef$; $a^2 = \frac{1}{4}e^2 + \frac{1}{4}f^2 - \frac{1}{2}ef\cos\varphi$,
$b^2 = \frac{1}{4}e^2 + \frac{1}{4}f^2 + \frac{1}{2}ef\cos\varphi$; $a = b = 325$,
$\alpha = 12^0\,43'\,9'',6$, $\alpha_1 = \alpha_2 = 6^0\,21'\,34'',8$,
$\beta_1 = \beta_2 = 83^0\,38'\,25'',2$, $\varphi = 90^0$.

4. $f = 2F : e\sin\varphi$, a und b wie vorher. $a = 173{,}858$,
$b = 174{,}544$, $f = 84{,}5$, $\alpha = 151^0\,55'\,42'',0$,
$\alpha_2 = 75^0\,30'\,46'',5$, $\alpha_1 = 76^0\,24'\,55'',5$, $\beta_1 = 14^0\,0'\,27'',5$,
$\beta_2 = 14^0\,3'\,50'',5$.

5. $(e+f)^2 = 2(b^2 + a^2) + 2(b^2 - a^2) : \cos\varphi$,
$(e-f)^2 = 2(b^2 + a^2) - 2(b^2 - a^2) : \cos\varphi$,
$\sin\alpha_2 = f\sin\varphi : 2a$, $\sin\alpha_1 = f\sin\varphi : 2b$,
$\alpha = \alpha_1 + \alpha_2$, u. s. w. $e = f = 377$, $F = 52440$,
$\alpha = 90^0$, $\alpha_2 = \beta_1 = 66^0\,13'\,21'',7$, $\alpha_1 = \beta_2 = 23^0\,46'\,38'',3$.

6.
$$b = \frac{a}{\sin\varphi}\cdot\sqrt{1 + \cos\varphi^2 - 2\cos\alpha^2\cos\varphi^2 + 2\sin\alpha\cos\varphi\sqrt{1 - \cos\alpha^2\cos\varphi^2}},$$
$\alpha_1 + \beta_2 = \varphi$, $\cos[\alpha - (\alpha_1 - \beta_2)] = \cos\alpha\cos\varphi$, u. s. w.
$b = 545$, $e = 663$, $F = 309852$, $\alpha_2 = 50^0\,41'\,32'',5$,
$\alpha_1 = 59^0\,2'\,21'',3$, $\beta_1 = 33^0\,3'\,52'',7$, $\beta_2 = 37^0\,12'\,13'',5$.

7. $\sin\alpha_2 = F : ae$, $b^2 = a^2 + e^2 - 2ae\cos\alpha_2$. (Zahlenbeispiel eindeutig.) $b = 23$, $f = 32{,}5269$, $\alpha = 90^0$,
$\alpha_2 = \alpha_1 = \beta_1 = \beta_2 = 45^0$, $\varphi = 90^0$.

8. $(e+f)^2 = 4a^2 + 4F\operatorname{cotg}\frac{1}{2}\varphi$, $(e-f)^2 = 4a^2 - 4F\operatorname{tang}\frac{1}{2}\varphi$,
$b^2 = a^2 + 2F\operatorname{cotg}\varphi$; $b = 357$, $e = f = 365$, $\alpha = 90^0$,
$\alpha_2 = 77^0\,58'\,55'',1 = \beta_1$, $\alpha_1 = \beta_2 = 12^0\,1'\,4'',9$.

9. $(a+e)^2 = b^2 + 2F\operatorname{cotg}\frac{1}{2}\alpha_2$, $(a-e)^2 = b^2 - 2F\operatorname{tang}\frac{1}{2}\alpha_2$;
$a = 136$, $e = f = 305$, $\alpha = 90^0$, $\alpha_1 = \beta_2 = 26^0\,28'\,51'',7$,
$\beta_1 = 63^0\,31'\,8'',3$, $\varphi = 52^0\,57'\,43'',4$.

10. $f^4(1 + 8\sin\beta_2{}^2) + 2f^2[4\sin\beta_2{}^2(e^2 - 2a^2) - e^2 - 4a^2]$
$+ (e^2 - 4a^2)^2 = 0$; $b = 89$, $f = 78$, $F = 6240$,
$\alpha = 51^0\,58'\,42'',4$, $\alpha_2 = \alpha_1 = 25^0\,59'\,21'',2$,
$\beta_1 = \beta_2 = 64^0\,0'\,38'',8$.

11. $(a+b)^2 = \frac{e^2 \cos \frac{1}{2}\alpha^2 - f^2 \sin \frac{1}{2}\alpha^2}{\cos \alpha}$,

$(a-b)^2 = \frac{f^2 \cos \frac{1}{2}\alpha^2 - e^2 \sin \frac{1}{2}\alpha^2}{\cos \alpha}$,

$\operatorname{tang} \frac{1}{2}(\beta_2 - \beta_1) = \sqrt{\frac{f^2 \operatorname{cotg} \frac{1}{2}\alpha^2 - e^2}{e^2 - f^2 \operatorname{tang} \frac{1}{2}\alpha^2}}$; $a = b = 85$, $F = 5544$.

$\alpha_2 = \alpha_1 = 25^0\,3'\,27'',4$, $\beta_1 = \beta_2 = 64^0\,56'\,32'',6$, $\varphi = 90^0$.

12. $\sin \frac{1}{2}(\alpha_2 - \alpha_1) = \frac{(a-b)\sin\frac{1}{2}\alpha}{e}$, $a + b = \frac{e \cos \frac{1}{2}(\alpha_2 - \alpha_1)}{\cos \frac{1}{2}\alpha}$,

$\alpha_2 + \alpha_1 = \alpha$; $a = 964$, $b = 773$, $f = 1723{,}07$,

$F = 187980$, $\alpha_2 = 42^0\,4'\,30'',1$, $\alpha_1 = 123^0\,18'\,48'',4$,

$\beta_1 = 6^0\,29'\,53'',1$, $\beta_2 = 8^0\,6'\,48'',4$.

13. $b + a = \frac{1}{2}u$,

$b - a = u\,(\sqrt{\sin \alpha^2 + \operatorname{tang} \varphi^2} - \operatorname{tang} \varphi) : 2 \sin \alpha$;

$a = 532$, $b = 629$, $e = 435$, $f = 1080{,}79$,

$F = 228228$, $\alpha_2 = 80^0\,28'\,21'',8$, $\alpha_1 = 56^0\,31'\,27'',9$,

$\beta_1 = 23^0\,23'\,12'',4$, $\beta_2 = 19^0\,36'\,57'',9$.

b) 1. $\cos \alpha = \frac{a-c}{2b}$, $\operatorname{tang} \alpha_2 = \frac{2b \sin \alpha}{a+c}$,

$e^2 = b^2 \sin \alpha^2 + \frac{1}{4}(a+c)^2$, $F = \frac{1}{2}(a+c)\,b \sin \alpha$.

2. $b = \frac{a-c}{2\cos\alpha}$, $4e^2 = (a-c)^2 \operatorname{tang} \alpha^2 + (a+c)^2$,

$\operatorname{tang} \alpha_2 = \frac{a-c}{a+c} \operatorname{tang} \alpha$, $F = \frac{1}{4}(a^2 - c^2) \operatorname{tang} \alpha$.

3. $b = \frac{d}{2\cos\alpha}$, $\sin \alpha_2 = \frac{d}{2e} \operatorname{tang} \alpha$, $a + c = 2e \cos \alpha_2$,

$F = \frac{1}{2} e d \operatorname{tang} \alpha \cos \alpha_2$.

4. $\operatorname{tang} \alpha = \frac{2h}{d}$, $b^2 = h^2 + \frac{1}{4}d^2$, $\sin \alpha_2 = \frac{h}{e}$,

$a + c = 2e \cos \alpha_2$, $F = he \cos \alpha_2$.

5. $\cos \alpha_2 = \frac{a+c}{2e}$, $h = e \sin \alpha_2$, $\operatorname{tang} \alpha = \frac{2h}{a-c}$,

$b = \frac{h}{\sin\alpha}$, $F = \frac{1}{2}(a+c)\,h$.

6. $e = \frac{a+c}{2\cos\alpha_2}$, $\operatorname{tang} \alpha = \frac{2e \sin \alpha_2}{a-c}$, $b = \frac{a-c}{2\cos\alpha}$,

$F = \frac{1}{2}(a+c)\,e \sin \alpha_2$.

7. $\cos \alpha_2 = \frac{s}{2e}$, $a - c = 2e \operatorname{cotg} \alpha \sin \alpha_2$, $b = \frac{e \sin \alpha_2}{\sin \alpha}$,

$F = \frac{1}{2} se \sin \alpha_2$.

8. $\operatorname{tang} \alpha_2 = \frac{h^2}{F}$, $e^2 = h^2 + \frac{F^2}{h^2}$, $\sin \alpha = \frac{h}{b}$, $a + c = \frac{2F}{h}$, $a - c = 2b \cos \alpha$.

9. $\sin \gamma_1 = \frac{a \sin \alpha_1}{c}$, $\alpha_2 = 90^0 - \frac{1}{2}(\alpha_1 + \gamma_1)$, $\alpha = \alpha_1 + \alpha_2$, $e = \frac{a+c}{2 \cos \alpha_2}$, $b = \frac{a-c}{2 \cos \alpha}$, $F = \frac{1}{4}(a + c)^2 \operatorname{tang} \alpha_2$.

10. $a^2 = c^2 + 4ch \operatorname{cotg} \alpha_1 - 4h^2$, $\operatorname{tang} \alpha = 2h : (a - c)$. Vergl. **b)** 2.

Zahlenbeispiele zu **b) 1—10.**

a	b	c	e	F	α	α_1	α_2	γ_1	h
1052	269	532	795	54648	14°51′46″,2	9°53′1″,5	4°58′44″,7	160° 9′29″,1	69
244	197	188	291	42120	81.49.43,6	39.45.13,5	42. 4.30,1	56. 5.46,3	195
668	221	388	555	90288	50.41.32,5	32.44.49,6	17.56.42,9	111.21.44,6	171
1244	317	628	939	70200	13.41. 8,0	9. 6.15,6	4.34.52,4	161.43.59,6	75
484	123	244	365	9828	12.40.49,4	8.26.17,5	4.14.31,9	163. 4.38,7	27

c) 1. $\cos \alpha = \frac{d^2 + (a-c)^2 - b^2}{2d(a-c)}$, $\cos \beta = \frac{b^2 + (a-c)^2 - d^2}{2b(a-c)}$,
$e^2 = \frac{(a^2 - b^2)c - (c^2 - d^2)a}{a - c}$, $f^2 = \frac{(a^2 - d^2)c - (c^2 - b^2)a}{a - c}$, etc.

2. $\sin(\gamma - \alpha) = \frac{a-c}{b} \sin \alpha$, $\beta = 180^0 - \gamma$, $d = \frac{b \sin \beta}{\sin \alpha}$, $F = \frac{1}{2}(a + c) b \sin \beta$.

3. $\sin \beta = \frac{h}{b}$, $\sin \alpha_2 = \frac{h}{e}$, $a = \sqrt{e^2 - h^2} + \sqrt{b^2 - h^2}$,
$d^2 = b^2 + (a - c)^2 - 2(a - c)\sqrt{b^2 - h^2}$, $\sin \alpha = h : d$.

4. $\sin \beta_1 = \frac{e \sin \varphi}{a + c}$, $\alpha_2 = \varphi - \beta_1$, $f = \frac{a+c}{\sin \varphi} \sin \alpha_2$,
$d^2 = e^2 + c^2 - 2ec \cos \alpha_2$, $b^2 = a^2 + e^2 - 2ae \cos \alpha_2$.

5. $\cos \beta_1 = \frac{f^2 + s^2 - e^2}{2fs}$, $\cos \alpha_2 = \frac{e^2 + s^2 - f^2}{2es}$,
$\sin \beta = \frac{e}{b} \sin \alpha_2$, $a = \frac{b \sin(\beta + \alpha_2)}{\sin \alpha_2}$.

6. $a = \frac{2F}{h} - c$, $f = \frac{h}{\sin \beta_1}$, $\operatorname{tang} \alpha_2 = \frac{h^2}{2F - h^2 \operatorname{cotg} \beta_1}$,
$e = \frac{h}{\sin \alpha_2}$, $b^2 = a^2 + e^2 - 2ae \cos \alpha_2$,
$d^2 = a^2 + f^2 - 2af \cos \beta_1$.

7. $b = \frac{(a-c)\sin\alpha}{\sin(\alpha+\beta)}$, $d = \frac{(a-c)\sin\beta}{\sin(\alpha-\beta)}$,
$f^2 = a^2 + d^2 - 2ad\cos\alpha$.

8. $\alpha_2 = \varphi - \beta_1$, $e = \frac{(a+c)\sin\beta_1}{\sin\varphi}$, $f = \frac{(a+c)\sin\alpha_2}{\sin\varphi}$,
$b^2 = a^2 + e^2 - 2ae\cos\alpha_2$, $d^2 = a^2 + f^2 - 2af\cos\beta_1$.

9. $\cos\beta = \frac{a^2+b^2-c^2}{2ab}$, $\cos\alpha_2 = \frac{a^2+e^2-b^2}{2ae}$,
$\operatorname{tang}\frac{1}{2}\delta = \frac{e\sin\alpha_2}{s - e\cos\alpha_2}$, $d = \frac{e\sin\alpha_2}{\sin\delta}$, $c = \frac{e\sin(\delta+\alpha_2)}{\sin\delta}$.

10. $\sin\frac{1}{2}(\alpha+\beta) = \frac{a-c}{b-d}\sin\frac{1}{2}(\alpha-\beta)$,
$b + d = (a-c)\cdot\frac{\cos\frac{1}{2}(\alpha-\beta)}{\cos\frac{1}{2}(\alpha+\beta)}$, $\sin\beta_1 = \frac{d}{e}\sin\alpha$,
$a = \frac{e}{\sin\alpha}\sin(\alpha+\beta_1)$.

11. $k = s : 4\cos\frac{1}{2}\alpha \, . \, \cos\frac{1}{2}\beta \, . \, \sin\frac{1}{2}(\alpha+\beta)$, $b = k\sin\alpha$,
$d = k\sin\beta$, $a - c = k\sin(\alpha+\beta)$,
$\sin\beta_1 = d \, . \, \sin\alpha : e$, $a = e \, . \, \sin(\alpha+\beta_1) : \sin\alpha$.

12. $\varphi - \beta_1 = \alpha_2$, $c = \frac{s\cos\frac{1}{2}\varphi}{\cos\frac{1}{2}(\alpha_2-\beta_1)} - a$, $e = \frac{a\sin\beta_1}{\sin\varphi}$,
$f = \frac{a\sin\alpha_2}{\sin\varphi}$, $b^2 = a^2 + e^2 - 2ae\cos\alpha_2$.

13. $\beta = 180^0 - \gamma$, $b = \frac{h}{\sin\gamma}$, $d = \frac{h}{\sin\alpha}$,
$a - c = \frac{h\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$, $a + c = u - b - d$.

Zahlenbeispiele zu c) 1—13.

a	b	c	d	e	f	F	α	β
428	289	260	257	307,936	471	87720	82° 50′ 50″,4	61° 55′ 39″,1
268	255	0	281	281	255	30954	55. 17. 31,0	64. 56. 32,6
1004	223,395	696	305	943	807	175950	42. 44. 28,5	67. 54. 46,7
436	153	12	377	388,23	159	30240	20. 58. 58,6	61. 55. 39,1
1676	145	1144	425	1562,4	1263	122670	12. 48. 44,2	36. 52. 11,6

α_2	β_1
41° 7′ 50″,5	32° 46′ 44″,7
55. 17. 31,0	64. 56. 32,6
12. 40. 49,4	14. 51. 46,2
20. 20. 55,4	58. 6. 33,2
3. 11. 31,6	3. 56. 59,6

d) **1.** $\sin\gamma_1 = a : 2r$, $\sin\delta_1 = b : 2r$, $\sin\alpha_1 = c : 2r$,
$\delta = \delta_1 + \gamma_1$, $\alpha = \alpha_1 + \delta_1$, $\beta = 180^0 - \delta$,
$\gamma = 180^0 - \alpha$, $\beta_1 = \beta - \alpha_1$, $d = 2r\sin\beta_1$,
$e = 2r\sin\beta$, $f = 2r\sin\alpha$, $F = \frac{1}{2}(ab + cd)\sin\beta$.

2. $\sin\gamma_1 = a : 2r$, $\sin\delta_1 = b : 2r$, $\alpha_1 = \alpha - \delta_1$,
$c = 2r\sin\alpha_1$, $\gamma = 180^0 - \alpha$, $\beta_1 = \gamma - \gamma_1$,
$d = 2r\sin\beta_1$, $\beta = \alpha_1 + \beta_1$, etc.

3. $\sin\gamma_1 = a : 2r$, $\sin\alpha_1 = c : 2r$, $\delta_1 = \alpha - \alpha_1$,
$b = 2r\sin\delta_1$, $\gamma = 180^0 - \alpha$, $\beta_1 = \gamma - \gamma_1$,
$d = 2r\sin\beta_1$.

4. $\sin\gamma_1 = a : 2r$, $\sin\alpha_1 = c : 2r$, $2\beta_1 = \sigma - \alpha_1 - \gamma_1$,
$\beta = \alpha_1 + \beta_1$, $\gamma = \beta_1 + \gamma_1$, $\alpha = 180^0 - \gamma$,
$\delta = 180^0 - \beta$, $d = 2r\sin\beta_1$, $b = 2r\sin\delta_1$.

5. $\sin\alpha = f : 2r = \sin\gamma$, $\sin\beta = e : 2r = \sin\delta$,
$2\gamma_1 = \gamma + \delta - \varphi$.

6. $\delta = 180^0 - \beta$, $\gamma_1 + \delta_1 = \delta$,
$\operatorname{tang}\frac{1}{2}(\gamma_1 - \delta_1) = (a - b)\operatorname{cotg}\frac{1}{2}\beta : (a + b)$.

7. $\cos\beta = \frac{a^2 + b^2 - e^2}{2ab}$, $\cos\gamma_1 = \frac{b^2 + e^2 - a^2}{2be}$,
$\cos\delta_1 = \frac{a^2 + e^2 - b^2}{2ae}$, $\sin\alpha = \frac{f\sin\gamma_1}{a}$, $\alpha_1 = \alpha - \delta_1$,
$\beta_1 = \beta - \alpha_1$, $d = a\sin\beta_1 : \sin\gamma_1$.

8. $\delta = 180^0 - \beta$, $\sin\gamma_1 = a\sin\beta : e$, $\sin\alpha_1 = c\sin\beta : e$,
$\beta_1 = \beta - \alpha_1$, $\gamma = \beta_1 + \gamma_1$.

9. $f_2 = \frac{e_1 \cdot e_2}{f_1}$, $\cos\beta_1 = \frac{a^2 + f_1^2 - e_1^2}{af_1}$,
$\cos\delta_1 = \frac{a^2 + e_1^2 - f_1^2}{ae_1}$, $c = \frac{e_2\sin(\beta_1 + \delta_1)}{\sin\delta_1}$,
$b^2 = f_1^2 + e_2^2 - 2f_1e_2\cos(\beta_1 + \delta_1)$,
$d^2 = e_1^2 + f_2^2 - 2e_1f_2\cos(\beta_1 + \delta_1)$.

10. $CE = q = -\frac{1}{2}c + \sqrt{p(a + p) + \frac{1}{4}c^2}$,
$\cos\beta = \frac{q^2 - b^2 - p^2}{2bp}$, $\cos\gamma = \frac{p^2 - b^2 - q^2}{2bq}$,
$\alpha = 180^0 - \gamma$, $\delta = 180^0 - \beta$,
$d = (a + p)\sin(\alpha + \delta) : \sin\delta$.

11. $d = \frac{p^2 - ac}{b}$, $\cos\beta = \frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}$,
$\cos\alpha = \frac{a^2 + d^2 - b^2 - c^2}{2(ad + bc)}$.

12. $f_2 = \frac{e_1 \cdot e_2}{f_1}$, $\sin\alpha = \frac{f}{e}\sin\delta$, $\tang\eta = \frac{e_2+e_1}{e_2-e_1}\cotg\delta$,

$\tang\vartheta = \frac{f_2+f_1}{f_2-f_1}\cotg\alpha$, $\varphi = \eta + \vartheta$,

$a^2 = e_1^2 + f_1^2 + 2e_1f_1\cos\varphi$, $b^2 = e_2^2 + f_1^2 - 2e_2f_1\cos\varphi$.

Zahlenbeispiele zu d) 1—12.

(Sind dieselben in Folge der Bestimmung durch Sinus zwei- oder mehrdeutig, so ist in dem Folgenden nur eins der Vierecke angegeben.)

a	b	c	d	e	f	F	α
56	33	16	63	65	45,769	1428	44° 45′ 36″,9
84	13	36	77	85	47,353	1932	33. 51. 18,1
14	13	13	4	15	15,6	108	106. 15. 36,8
180	299	180	299	349	349	53820	90.
3,72350	6,86267	9,69660	2,60735	9	6	23,383	142. 10. 22,2

β	α_1	β_1	γ_1
90°	14° 15′ 0″,1	75° 44′ 59″,9	59° 29′ 23″,2
90°	25. 3. 27,4	64. 56. 32,6	81. 12. 9,3
67° 22′ 48″,5	53. 7. 48,4	14. 15. 0,1	59. 29. 23,1
90.	31. 2. 53,6	58. 57. 6,4	31. 2. 53,6
113. 5. 10,0	97. 37. 46,1	15. 27. 23,9	22. 22. 13,9

δ_1	r
30° 30′ 36″,8	32,5
8. 47. 50,7	42,5
53. 7. 48,4	8,125
58. 57. 6,4	$e_1 = e_2 = f_1 = f_2 = 174{,}5$
44. 32. 36,1	$e_1 = 1$, $e_2 = 8$, $f_1 = 4$, $f_2 = 2$, $\varphi = 60^0$.

e) 1. $\sin\gamma_1 = \frac{a}{e}\sin\beta$, $\alpha_2 = 180^0 - \beta - \gamma_1$,

$b = \frac{e}{\sin\beta}\sin\alpha_2$, $\cotg\frac{1}{2}\gamma = \frac{b}{\varrho} - \cotg\frac{1}{2}\beta$,

$\cotg\frac{1}{2}\alpha = \frac{a}{\varrho} - \cotg\frac{1}{2}\beta$, etc. $b = c = d = 65$,

$f = 66$, $\alpha = 61^0 1' 13'',6$, $\gamma_1 = \gamma_2 = \alpha_1 = \alpha_2 = 30^0 30' 36'',8$,

$F = 3696$.

2. $\cotg\frac{1}{2}\alpha = \frac{a}{\varrho} - \cotg\frac{1}{2}\beta$, $\cotg\frac{1}{2}\gamma = \frac{b}{\varrho} - \cotg\frac{1}{2}\beta$,

$c = \frac{\varrho \sin \frac{1}{2}(\gamma + \delta)}{\sin \frac{1}{2}\gamma \sin \frac{1}{2}\delta}$, $d = \frac{\varrho \sin \frac{1}{2}(\alpha + \delta)}{\sin \frac{1}{2}\alpha \sin \frac{1}{2}\delta}$, $c = 28$,
$d = 75{,}6$, $F = 2822{,}4$, $\alpha = 45^0\ 14'\ 23'',0$,
$\gamma = 134^0\ 45'\ 37'',0$, $\delta = 106^0\ 15'\ 36'',8$.

3. $\cos \beta = \frac{a^2 + b^2 - e^2}{2ab}$, $d = a + c - b$,
$\cos \gamma = \frac{c^2 + d^2 - e^2}{2cd}$; $d = 10$, $\alpha = \gamma = 90^0$, $\beta = 120^0$,
$\delta = 60^0$, $F = 57{,}735$.

4. $\cos \beta = \frac{a^2 + b^2 - e^2}{2ab}$, $\varrho = \frac{a \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta}{\sin \frac{1}{2}(\alpha + \beta)}$,
$\operatorname{cotg} \frac{1}{2}\gamma = \frac{b}{\varrho} - \operatorname{cotg} \frac{1}{2}\beta$, $c = \frac{\varrho \sin \frac{1}{2}(\gamma + \delta)}{\sin \frac{1}{2}\gamma \sin \frac{1}{2}\delta}$,
$a = 75$, $b = 58$, $c = 24$, $d = 41$, $\varrho = 20$, $F = 1980$,
$\alpha = 77^0\ 19'\ 10'',6$, $\beta = 43^0\ 36'\ 10'',1$, $\gamma = 136^0 23'\ 49'',9$,
$\delta = 102^0\ 40'\ 49'',4$.

5. $\varrho = \frac{a \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta}{\sin \frac{1}{2}(\alpha + \beta)}$, $\operatorname{cotg} \frac{1}{2}\gamma = \frac{b}{\varrho} - \operatorname{cotg} \frac{1}{2}\beta$,
$c = \frac{\varrho \sin \frac{1}{2}(\gamma + \delta)}{\sin \frac{1}{2}\gamma \sin \frac{1}{2}\delta}$; $c = 102$, $d = 229{,}732$, $\varrho = 60$,
$F = 18223{,}92$, $\gamma = 159^0\ 13'\ 20'',0$, $\delta = 66^0\ 47'\ 49'',2$.

6. $\sin \alpha = 2\varrho : d$, $\sin \beta = 2\varrho : b$, $\gamma = 180^0 - \beta$,
$\delta = 180^0 - \alpha$, $a + c = b + d$,
$a - c = d \sin(\alpha + \beta) : \sin \beta$, $F = (b + d)\,\varrho$;
$a = 1083$, $c = 135$, $F = 173565$, $\alpha = 17^0\ 56'\ 42'',9$;
$\beta = 76^0\ 34'\ 49'',2$, $\gamma = 103^0 25'\ 10'',8$, $\delta = 162^0 3'\ 17'',1$.

7. $d = s - b$, $\sin \alpha = 2\varrho : d$, $\sin \beta = 2\varrho : b$,
$a - c = d \sin(\alpha + \beta) : \sin \beta$: $a = 241$, $c = 54$,
$d = 195$, $F = 14652$, $\alpha = 30^0\ 30'\ 36'',8$,
$\beta = 78^0 34' 43'',7$, $\gamma = 101^0 25' 16'',3$, $\delta = 149^0 29' 23'',2$.

8. $\sin \beta = \frac{2\varrho}{b}$, $\gamma = 180^0 - \beta$, $\operatorname{cotg} \frac{1}{2}\delta = \frac{c}{\varrho} - \operatorname{cotg} \frac{1}{2}\gamma$,
$\alpha = 180^0 - \delta$, $d = \frac{2\varrho}{\sin \alpha}$, $a = \frac{\varrho \sin \frac{1}{2}(\alpha + \beta)}{\sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta}$.
Zahlenbeispiel, vergl. 5.

f) 1. $\gamma = 180^0 - \alpha$, $\delta = 180^0 - \beta$, $\varrho = \frac{a \sin \frac{1}{2}\alpha \sin \frac{1}{2}\beta}{\sin \frac{1}{2}(\alpha + \beta)}$,
$b = \frac{\varrho \sin \frac{1}{2}(\beta + \gamma)}{\sin \frac{1}{2}\beta \sin \frac{1}{2}\gamma}$, etc.

2. $\gamma = 180^0 - \alpha$, $\operatorname{cotg} \frac{1}{2}\beta = \frac{a}{\varrho} - \operatorname{cotg} \frac{1}{2}\alpha$, $\delta = 180^0 - \beta$,
b wie vorher.

3. $\alpha = 180^0 - \gamma$. Vergl. 2.

4. $c = \frac{F}{\varrho} - a$. Setzt man $\frac{2\varrho}{\sqrt{ac}} = \sin\varphi$, so ist

$\operatorname{cotg}\frac{1}{2}\alpha = \operatorname{tang}\frac{1}{2}\gamma = \frac{a}{\varrho}\cos\frac{1}{2}\varphi^2$,

$\operatorname{cotg}\frac{1}{2}\beta = \operatorname{tang}\frac{1}{2}\delta = \frac{a}{\varrho}\sin\frac{1}{2}\varphi^2$.

Zahlenbeispiele zu f) 1—4.

ϱ	b	c	d	a	F	α	β
28	240	49,0205	21,4427	212,4222	7320,4	116° 13′ 6″,4	16° 20′ 32″,8
120	378	140	210	448	70560	73. 44. 23,2	45. 14. 23,0.

g) 1. $\sin\frac{1}{2}\beta = \frac{e}{2a}$, $\operatorname{cotg}\frac{1}{2}\alpha = \frac{a}{\varrho} - \operatorname{cotg}\frac{1}{2}\beta$,

$c = \frac{e}{2\sin(\alpha + \frac{1}{2}\beta)}$, etc, $a = 29$, $c = 75$, $e = 42$,
$f = 92$, $F = 1932$, $\alpha = 117^0 20' 33'',4$, $\beta = 92^0 47' 39'',8$,
$\delta = 32^0\ 31'\ 13'',4$.

2. $a = \frac{s}{1 + 2\sin\frac{1}{2}\beta}$, $\operatorname{tang}\frac{1}{2}\delta = \frac{e}{2(f - a\cos\frac{1}{2}\beta)}$,
$c^2 = \frac{1}{4}e^2 + (f - a\cos\frac{1}{2}\beta)^2$; $a = 689$, $c = 2055$,
$e = 222$, $f = 2732$, $F = 303252$, $\alpha = 167^0\ 37' 57'',9$,
$\delta = 6^0\ 11'\ 33'',4$.

3. $\sin\frac{1}{2}\beta = \frac{e'}{a}$, $\operatorname{tang}\frac{1}{2}\delta = \frac{e'}{2f' - a\cos\frac{1}{2}\beta}$,
$c^2 = e'^2 + (2f' - a\cos\frac{1}{2}\beta)^2$; $c = 569$, $e = 462$,
$f = 628$, $F = 145068$, $\alpha = 91^0 6' 19'',3$, $\beta = 129^0 53' 5'',2$,
$\delta = 47^0\ 54'\ 16'',2$.

4. $\operatorname{tang}\frac{1}{2}\beta = \frac{a \pm \sqrt{a^2 - 4p_a{}^2}}{2p_a}$, $e = 2a\sin\frac{1}{2}\beta$,

$\cos\frac{1}{2}\delta = \frac{2p_c}{e}$; $c_1 = 70{,}41667$, $e = 130$, $f = 137{,}3864$,
$F = 8930{,}1$, $\alpha = 82^0\ 6'\ 34'',7$, $\beta = 61^0\ 1'\ 13'',6$,
$\delta = 134^0\ 45'\ 37'',0$.

5. $c = \frac{d''}{2\sin\frac{1}{2}\delta - 1}$, $e = \frac{2d''\sin\frac{1}{2}\delta}{2\sin\frac{1}{2}\delta - 1}$, $a = e - d''$,

$\sin\frac{1}{2}\beta = \frac{e}{2a}$; $a = 375$, $c = 449$, $e = 702$, $f = 412$,

$F = 144612$, $\alpha = 59^0\ 11'\ 23'',2$, $\beta = 138^0\ 46'\ 50'',2$, $\delta = 102^0\ 50'\ 23'',4$.

6. $a = \frac{s(2\sin\frac{1}{2}\beta + \cos\frac{1}{2}\beta) \pm \sqrt{c^2(2\sin\frac{1}{2}\beta + \cos\frac{1}{2}\beta)^2 + (c^2 - s^2)\sin\frac{1}{2}\beta^2}}{(2\sin\frac{1}{2}\beta + \cos\frac{1}{2}\beta)^2 + \sin\frac{1}{2}\beta^2}$, $e = 2a\sin\frac{1}{2}\beta$, $\sin\frac{1}{2}\delta = a\sin\frac{1}{2}\beta : c$, Beispiel, siehe g) 1.

7. $e = 2a\sin\frac{1}{2}\beta$, $f = \frac{n}{m}e$, $\operatorname{tang}\frac{1}{2}\delta = \frac{a\sin\frac{1}{2}\beta}{f - a\cos\frac{1}{2}\beta}$, $c = \frac{e}{2\sin\frac{1}{2}\delta}$. Siehe g) 2.

8. $e = \sqrt{\frac{2m}{n}F}$, $f = \sqrt{\frac{2n}{m}F}$, $\sin\frac{1}{2}\beta = \frac{e}{2a}$, $\operatorname{tang}\frac{1}{2}\delta = \frac{e}{2(f - a\cos\frac{1}{2}\beta)}$. Siehe g) 5.

§. 30.

A) 1. $e = 16,15$, $\delta = 65^0\ 34'\ 51''$, $\varepsilon = 130^0\ 43'\ 9''$.

2. $96^0\ 30'\ 28''$; 1,0478; 6,8273.

3. $AB = 15$, $BC = 13$, $CD = 37$, $DE = 17$, $EF = 36,0942$, $FG = 15$, $GA = 40$, $\sphericalangle A = 126^0\ 52'\ 11'',6$, $B = 120^0\ 30'\ 36'',9$, $C = 167^0\ 26'\ 6'',0$, $D = 80^0\ 51'\ 7'',8$, $E = 167^0\ 4'\ 17'',6$, $F = 110^0\ 23'\ 28'',4$, $G = 126^0\ 52'\ 11'',6$, $F = 1933,93$.

4. $AC = 884$, $BC = 116$, $BD = 404$, $CD = 480$, $CE = 106$, $DE = 394$, $CF = 123$, $EF = 65$, $EG = 93$, $FG = 34$.

5. $DC = 442$, $\sphericalangle EDC = 30^0 22' 42'',8$, $\sphericalangle EAB = 239^0 37' 17'',2$, $F = 65580$.

6. $a = -2r\sin(\gamma + \varepsilon)$, $b = -2r\sin(\alpha + \delta)$,
$c = -2r\sin(\beta + \varepsilon)$, $d = -2r\sin(\alpha + \gamma)$,
$e = -2r\sin(\beta + \delta)$,

$$F = 2r^2 . [\sin\gamma . \sin(\alpha + \delta) . \sin(\beta + \varepsilon) + \sin\varepsilon . \sin(\alpha + \gamma) . \sin(\beta + \delta) - \sin\gamma . \sin\varepsilon . \sin(\gamma + \varepsilon)].$$

7. $C = 61^0\ 55'\ 40'',5$ (39,1), $A = 28^0\ 4'\ 19'',5$ (20,9), $AE = 93$, $ED = 123$, $DC = 102$, $EB = 65$, $BD = 106$, $F = 8868$.

Anhang 5.

8. $\cos(2x - 45^0) = -\frac{1}{11}\sqrt{2}$. $x_1 = 70^0\ 23'\ 55'',9$.

9. $\sin 2x = \sqrt{5} - 2$; $x_1 = 13^0\ 21'\ 30''$.

10. $9 \operatorname{cotg} x^4 - 13 \operatorname{cotg} x^2 = 8$; $35^0\ 53'\ 21'',1$.

11. $36^0\ 18'\ 31''$; $159^0\ 38'\ 14''$. **12.** $\sin x = 0$ oder $\cos x = \frac{1}{2}$.

13. $\sin(x - y) = 0{,}1$, $\sin(x + y) = 0{,}3$, $x_1 = 11^0\ 35'\ 54''$, $y_1 = 5^0\ 51'\ 33''$. **14.** 45^0; $37^0\ 42'$.

15. $\operatorname{tang} \frac{1}{2}(x + y) = \pm 1$; $x_1 = 53^0\ 7'\ 48'',2$, $y_1 = 36^0\ 52'\ 11'',8$, $x_{11} = 143^0\ 7'\ 48'',2$, $y_{11} = 126^0\ 52'\ 11'',8$, etc.

16. 45^0; 180^0; 225^0; 270^0.

17. $\operatorname{cotg} \frac{1}{2}\beta = \frac{a - e}{e} = \operatorname{tang} \frac{1}{2}\alpha$; $\alpha = 44^0\ 24'\ 31''$, $\beta = 45^0\ 35'\ 29''$, $b = 404{,}74$, $c = 566{,}55$, $F = 80240$.

18. $c = \sqrt{\frac{1}{2}S}$, $a + b = s - c$, $a - b = \sqrt{c^2 + 2cs - s^2}$; $c = 55$, $a = 44$, $b = 33$, $F = \frac{1}{4}s\,(s - \sqrt{2S}) = 726$, $\alpha = 53^0\ 7'\ 48'',4$, $\beta = 36^0\ 52'\ 11'',6$.

19. $a = \frac{\sqrt{5} - 1}{2}\,s$, $b = \frac{3 - \sqrt{5}}{2}\,s$, $c = \sqrt{5 - 2\sqrt{5}}\,.\,s$, $F = \frac{\sqrt{5} - 2}{2}\,s^2$, $\alpha = 58^0\ 16'\ 57''$.

20. $c = \frac{5}{4}a$, $b = \frac{3}{4}a$, $F = \frac{3}{8}a^2$, $\alpha = 53^0\ 7'\ 48'',4$, $\beta = 36^0\ 52'\ 11'',6$.

21. $76^0\ 20'\ 44''$; $51^0\ 49'\ 38''$. **22.** $a(1 + \sin \frac{1}{2}\beta) : \pi = 10$.

23. $a = s\sqrt{2} : 4\cos(45^0 - \frac{1}{2}\alpha)$, $F = a^2 \sin \alpha$, $a = 9{,}594$, $F = 82{,}194$.

24. $a^2 = \frac{1}{2}[4m_a{}^2 - 3h^2 \pm \sqrt{9h^4 - 40h^2 m_a{}^2 + 16 m_a{}^4}]$, $b^2 = m^2 - \frac{1}{4}a^2$, $\sin \alpha^2 = [3h^2 + 4m^2 \pm \sqrt{9h^4 - 40h^2m^2 + 16m^4}] : 8m^2$.

25. $m \cdot \frac{1 + \cos\alpha + \cos\alpha\cos\beta}{1 - \cos\alpha\cos\beta\cos\gamma}$.

27. $(a - b)^2 = c^2 - (s^2 - c^2)\operatorname{tang} \frac{1}{2}\beta^2$, $a = 316$, $b = 305$, $F = 86268$.

28. $\sin \frac{1}{2}\alpha = \frac{2}{7}\pi$; $F\left(\frac{\alpha^0}{360} - \frac{\sin\alpha}{2\pi}\right) = 603{,}42$.

29. $a = p + q$, $\operatorname{tang} \varphi = \frac{a}{q \operatorname{cotg} w - p \operatorname{cotg} v}$, $b = \frac{q \sin \varphi}{\sin w}$,
$c = \frac{p \sin \varphi}{\sin v}$, $\beta = 180^0 - (\varphi + v)$, $\gamma = \varphi - w$, $\alpha = 86^0$,
$\varphi = 7^0\,50'\,3'',1$, $\beta = 172^0\,6'\,16'',9$, $\gamma = 7^0\,45'\,3'',1$,
$b = 4685,9$, $c = 4600,7$, $F = 27174,6$.

30. $\cos \gamma^2 \cdot (256 + 36\pi^2)\, a^2 b^2 - \cos \gamma \cdot 36\pi^2 \cdot ab \cdot (a^2 + b^2)$
$= 256 a^2 b^2 - 9\pi^2 (a^2 + b^2)^2$; $\gamma = 79^0\,30'\,42'',3$ od. $1^0\,8'\,29'',1$.

31. $h = \frac{2}{3} a \sqrt{3} = 3,46410$, $b = a\sqrt{3} = 5,19615$,
$c = \frac{5}{6} a \sqrt{3} = 4,33013$, $\alpha = 53^0\,7'\,48'',4$.

32. $F = \frac{5 d^2}{8 \sin 108^0} = 42,058$; $u = \frac{5d}{2 \sin 54^0} = 24,721$.

33. α) $124^0\,58'\,33'',6$; 2,9; 10,1; 126. β) $41^0\,6'\,43'',5$; 73; 55; 2814.

34. 184; $16^0\,15'\,36'',7$; $148^0\,35'\,20'',2$; 5460.

35. $1 - \sin 2\alpha : 1$.

36. $\sin 2\alpha = (n - m) : m = 1$, $\alpha = 45^0$. Die Eckpunkte des kleinsten halbiren die Seiten des gegebenen Quadrats.

37. $x^2 = 2a^2 + 2ab + b^2$, $y^2 = a^2 + 2ab + 2b^2$,
$\sin (x, a) = a : x$, $\sin (y, b) = b : y$,
$\sin (x, c) = a^2 : x\sqrt{a^2 + b^2}$,
$\sin (y, c) = b^2 : y\sqrt{a^2 + b^2}$.

38. $73^0\,44'\,23'',3$; $106^0\,15'\,36'',7$; 65; 62,401; 1428.

39. $c^2 + e^2 = d^2 + f^2$, $d^2 - e^2 = cd \cos \alpha - fe \cos \gamma$,
$d^2 - c^2 = de \cos \beta - cf \cos (\alpha + \beta + \gamma)$.

40. $a^2 = \frac{1}{2}[b^2 + e^2 \pm \sqrt{4 b^2 e^2 - (c^2 - d^2)^2}]$,
$c^2 + d^2 = b^2 + e^2$, $\sin (b, a) = (a^2 + b^2 - d^2) : 2ab$, etc.

41. $F = \frac{hk}{\sin \gamma}$, $\operatorname{tang} \varphi = \frac{h \sin \gamma}{k + h \cos \gamma}$, $\operatorname{tang} \psi = \frac{k \sin \gamma}{h + k \cos \gamma}$.

42. $\sin (2\varphi - \alpha) = 3 \sin \alpha$, $x = a \operatorname{tang} \varphi = 58,56$.

43. $29^{m},625$.

44. $\operatorname{tang} \alpha = \frac{h_1 + h_2}{\sqrt{d^2 - (h_2 - h_1)^2}}$, $\alpha = 54^0\,12'\,46'',7$;

$S_1 X = \frac{h_1}{\sin \alpha} = 716,62$, $S_2 X = \frac{h_2}{\sin \alpha} = 3092,9$.

45. $\alpha_1 = 180^0 - 2\alpha$, $\beta_1 = 180^0 - 2\beta$, $\gamma_1 = 180^0 - 2\gamma$,
$a_1 = 2a \cos \alpha$, $b_1 = 2b \cos \beta$, $c_1 = 2c \cos \gamma$,
$F_1 = 8F \cos \alpha \cos \beta \cos \gamma$.

7. $b = \frac{(a-c)\sin\alpha}{\sin(\alpha+\beta)}$, $d = \frac{(a-c)\sin\beta}{\sin(\alpha+\beta)}$,
$f^2 = a^2 + d^2 - 2ad\cos\alpha$.

8. $\alpha_2 = \varphi - \beta_1$, $e = \frac{(a+c)\sin\beta_1}{\sin\varphi}$, $f = \frac{(a+c)\sin\alpha_2}{\sin\varphi}$,
$b^2 = a^2 + e^2 - 2ae\cos\alpha_2$, $d^2 = a^2 + f^2 - 2af\cos\beta_1$.

9. $\cos\beta = \frac{a^2+b^2-e^2}{2ab}$, $\cos\alpha_2 = \frac{a^2+e^2-b^2}{2ae}$,
$\operatorname{tang}\frac{1}{2}\delta = \frac{e\sin\alpha_2}{s-e\cos\alpha_2}$, $d = \frac{e\sin\alpha_2}{\sin\delta}$, $c = \frac{e\sin(\delta+\alpha_2)}{\sin\delta}$.

10. $\sin\frac{1}{2}(\alpha+\beta) = \frac{a-c}{b-d}\sin\frac{1}{2}(\alpha-\beta)$,
$b + d = (a-c)\cdot\frac{\cos\frac{1}{2}(\alpha-\beta)}{\cos\frac{1}{2}(\alpha+\beta)}$, $\sin\beta_1 = \frac{d}{e}\sin\alpha$,
$a = \frac{e}{\sin\alpha}\sin(\alpha+\beta_1)$.

11. $k = s : 4\cos\frac{1}{2}\alpha \,.\, \cos\frac{1}{2}\beta \,.\, \sin\frac{1}{2}(\alpha+\beta)$, $b = k\sin\alpha$,
$d = k\sin\beta$, $a - c = k\sin(\alpha+\beta)$,
$\sin\beta_1 = d \,.\, \sin\alpha : e$, $a = e \,.\, \sin(\alpha+\beta_1) : \sin\alpha$.

12. $\varphi - \beta_1 = \alpha_2$, $c = \frac{s\cos\frac{1}{2}\varphi}{\cos\frac{1}{2}(\alpha_2-\beta_1)} - a$, $e = \frac{a\sin\beta_1}{\sin\varphi}$,
$f = \frac{a\sin\alpha_2}{\sin\varphi}$, $b^2 = a^2 + e^2 - 2ae\cos\alpha_2$.

13. $\beta = 180^0 - \gamma$, $b = \frac{h}{\sin\gamma}$, $d = \frac{h}{\sin\alpha}$,
$a - c = \frac{h\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$, $a + c = u - b - d$.

Zahlenbeispiele zu c) 1—13.

a	b	c	d	e	f	F	α	β
428	289	260	257	307,936	471	87720	82° 50′ 50″,4	61° 55′ 39″,1
268	255	0	281	281	255	30954	55. 17. 31,0	64. 56. 32,6
1004	223,395	696	305	943	807	175950	42. 44. 28,5	67. 54. 46,7
436	153	12	377	388,23	159	30240	20. 58. 58,6	61. 55. 39,1
1676	145	1144	425	1562,4	1263	122670	12. 48. 44,2	36. 52. 11,6

α_2	β_1
41° 7′ 50″,5	32° 46′ 44″,7
55. 17. 31,0	64. 56. 32,6
12. 40. 49,4	14. 51. 46,2
20. 20. 55,4	58. 6. 33,2
3. 11. 31,6	3. 56. 59,6

d) **1.** $\sin\gamma_1 = a : 2r$, $\sin\delta_1 = b : 2r$, $\sin\alpha_1 = c : 2r$,
$\delta = \delta_1 + \gamma_1$, $\alpha = \alpha_1 + \delta_1$, $\beta = 180^0 - \delta$,
$\gamma = 180^0 - \alpha$, $\beta_1 = \beta - \alpha_1$, $d = 2r \sin\beta_1$,
$e = 2r\sin\beta$, $f = 2r\sin\alpha$, $F = \frac{1}{2}(ab + cd)\sin\beta$.

2. $\sin\gamma_1 = a : 2r$, $\sin\delta_1 = b : 2r$, $\alpha_1 = \alpha - \delta_1$,
$c = 2r\sin\alpha_1$, $\gamma = 180^0 - \alpha$, $\beta_1 = \gamma - \gamma_1$,
$d = 2r\sin\beta_1$, $\beta = \alpha_1 + \beta_1$, etc.

3. $\sin\gamma_1 = a : 2r$, $\sin\alpha_1 = c : 2r$, $\delta_1 = \alpha - \alpha_1$,
$b = 2r\sin\delta_1$, $\gamma = 180^0 - \alpha$, $\beta_1 = \gamma - \gamma_1$,
$d = 2r\sin\beta_1$.

4. $\sin\gamma_1 = a : 2r$, $\sin\alpha_1 = c : 2r$, $2\beta_1 = \sigma - \alpha_1 - \gamma_1$,
$\beta = \alpha_1 + \beta_1$, $\gamma = \beta_1 + \gamma_1$, $\alpha = 180^0 - \gamma$,
$\delta = 180^0 - \beta$, $d = 2r\sin\beta_1$, $b = 2r\sin\delta_1$.

5. $\sin\alpha = f : 2r = \sin\gamma$, $\sin\beta = e : 2r = \sin\delta$,
$2\gamma_1 = \gamma + \delta - \varphi$.

6. $\delta = 180^0 - \beta$, $\gamma_1 + \delta_1 = \delta$,
$\operatorname{tang}\frac{1}{2}(\gamma_1 - \delta_1) = (a - b)\operatorname{cotg}\frac{1}{2}\beta : (a + b)$.

7. $\cos\beta = \frac{a^2 + b^2 - e^2}{2ab}$, $\cos\gamma_1 = \frac{b^2 + e^2 - a^2}{2be}$,
$\cos\delta_1 = \frac{a^2 + e^2 - b^2}{2ae}$, $\sin\alpha = \frac{f\sin\gamma_1}{a}$, $\alpha_1 = \alpha - \delta_1$,
$\beta_1 = \beta - \alpha_1$, $d = a\sin\beta_1 : \sin\gamma_1$.

8. $\delta = 180^0 - \beta$, $\sin\gamma_1 = a\sin\beta : e$, $\sin\alpha_1 = c\sin\beta : e$,
$\beta_1 = \beta - \alpha_1$, $\gamma = \beta_1 + \gamma_1$.

9. $f_2 = \frac{e_1 \cdot e_2}{f_1}$, $\cos\beta_1 = \frac{a^2 + f_1^2 - e_1^2}{af_1}$,
$\cos\delta_1 = \frac{a^2 + e_1^2 - f_1^2}{ae_1}$, $c = \frac{e_2\sin(\beta_1 + \delta_1)}{\sin\delta_1}$,
$b^2 = f_1^2 + e_2^2 - 2f_1 e_2\cos(\beta_1 + \delta_1)$,
$d^2 = e_1^2 + f_2^2 - 2e_1 f_2\cos(\beta_1 + \delta_1)$.

10. $CE = q = -\frac{1}{2}c + \sqrt{p(a + p) + \frac{1}{4}c^2}$,
$\cos\beta = \frac{q^2 - b^2 - p^2}{2bp}$, $\cos\gamma = \frac{p^2 - b^2 - q^2}{2bq}$,
$\alpha = 180^0 - \gamma$, $\delta = 180^0 - \beta$,
$d = (a + p)\sin(\alpha + \delta) : \sin\delta$.

11. $d = \frac{p^2 - ac}{b}$, $\cos\beta = \frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}$,
$\cos\alpha = \frac{a^2 + d^2 - b^2 - c^2}{2(ad + bc)}$.

11. $(a+b)^2 = \frac{e^2\cos\frac{1}{2}\varepsilon^2 - f^2\sin\frac{1}{2}\varepsilon^2}{\cos\varepsilon}$,

$(a-b)^2 = \frac{f^2\cos\frac{1}{2}\varepsilon^2 - e^2\sin\frac{1}{2}\varepsilon^2}{\cos\varepsilon}$,

$\operatorname{tang}\frac{1}{2}(\beta_2-\beta_1) = \sqrt{\frac{f^2\operatorname{tang}\frac{1}{2}\varepsilon^2 - e^2}{e^2 - f^2\operatorname{tang}\frac{1}{2}\varepsilon^2}}$; $a = b = 85$, $F = 5544$.

$\alpha_2 = \alpha_1 = 25^\circ\, 3'\, 27'',4$, $\beta_1 = \beta_2 = 64^\circ\, 56'\, 32'',6$, $\varphi = 90^\circ$.

12. $\sin\frac{1}{2}(\alpha_2 - \alpha_1) = \frac{(a-b)\sin\frac{1}{2}\alpha}{e}$, $a + b = \frac{e\cos\frac{1}{2}(\alpha_2-\alpha_1)}{\cos\frac{1}{2}\alpha}$,

$\alpha_2 + \alpha_1 = \alpha$; $a = 964$, $b = 773$, $f = 1723{,}07$,

$F = 187980$, $\alpha_2 = 42^\circ\, 4'\, 30'',1$, $\alpha_1 = 123^\circ\, 18'\, 48'',4$,

$\beta_1 = 6^\circ\, 29'\, 53'',1$, $\beta_2 = 8^\circ\, 6'\, 48'',4$.

13. $b + a = \frac{1}{2}u$,

$b - a = u\left(\sqrt{\sin\alpha^2 + \operatorname{tang}\varphi^2} - \operatorname{tang}\varphi\right) : 2\sin\alpha$;

$a = 532$, $b = 629$, $e = 435$, $f = 1080{,}79$,

$F = 228228$, $\alpha_2 = 80^\circ\, 28'\, 21'',8$. $\alpha_1 = 56^\circ\, 31'\, 27'',9$,

$\beta_1 = 23^\circ\, 23'\, 12'',4$, $\beta_2 = 19^\circ\, 36'\, 57'',9$.

b) 1. $\cos\alpha = \frac{a-c}{2b}$, $\operatorname{tang}\alpha_2 = \frac{2b\sin\alpha}{a+c}$,

$e^2 = b^2\sin\alpha^2 + \frac{1}{4}(a+c)^2$, $F = \frac{1}{2}(a+c)\,b\sin\alpha$.

2. $b = \frac{a-c}{2\cos\alpha}$, $4e^2 = (a-c)^2\operatorname{tang}\alpha^2 + (a+c)^2$,

$\operatorname{tang}\alpha_2 = \frac{a-c}{a+c}\operatorname{tang}\alpha$, $F = \frac{1}{4}(a^2 - c^2)\operatorname{tang}\alpha$.

3. $b = \frac{d}{2\cos\alpha}$, $\sin\alpha_2 = \frac{d}{2e}\operatorname{tang}\alpha$, $a + c = 2e\cos\alpha_2$,

$F = \frac{1}{2}ed\operatorname{tang}\alpha\cos\alpha_2$.

4. $\operatorname{tang}\alpha = \frac{2h}{d}$, $b^2 = h^2 + \frac{1}{4}d^2$, $\sin\alpha_2 = \frac{h}{e}$,

$a + c = 2e\cos\alpha_2$, $F = he\cos\alpha_2$.

5. $\cos\alpha_2 = \frac{a+c}{2e}$, $h = e\sin\alpha_2$, $\operatorname{tang}\alpha = \frac{2h}{a-c}$,

$b = \frac{h}{\sin\alpha}$, $F = \frac{1}{2}(a+c)\,h$.

6. $e = \frac{a+c}{2\cos\alpha_2}$, $\operatorname{tang}\alpha = \frac{2e\sin\alpha_2}{a-c}$, $b = \frac{a-c}{2\cos\alpha}$,

$F = \frac{1}{2}(a+c)\,e\sin\alpha_2$.

7. $\cos\alpha_2 = \frac{s}{2e}$, $a - c = 2e\operatorname{cotg}\alpha\sin\alpha_2$, $b = \frac{e\sin\alpha_2}{\sin\alpha}$,

$F = \frac{1}{2}se\sin\alpha_2$.

8. $\operatorname{tang} \alpha_2 = \frac{h^2}{F}$, $e^2 = h^2 + \frac{F^2}{h^2}$, $\sin \alpha = \frac{h}{b}$, $a + c = \frac{2F}{h}$, $a - c = 2b \cos \alpha$.

9. $\sin \gamma_1 = \frac{a \sin \alpha_1}{c}$, $\alpha_2 = 90^0 - \frac{1}{2}(\alpha_1 + \gamma_1)$, $\alpha = \alpha_1 + \alpha_2$, $e = \frac{a+c}{2 \cos \alpha_2}$, $b = \frac{a-c}{2 \cos \alpha}$, $F = \frac{1}{4}(a + c)^2 \operatorname{tang} \alpha_2$.

10. $a^2 = c^2 + 4ch \operatorname{cotg} \alpha_1 - 4h^2$, $\operatorname{tang} \alpha = 2h : (a - c)$. Vergl. **b)** 2.

Zahlenbeispiele zu b) 1—10.

a	b	c	e	F	α	α_1	α_2	γ_1	h
1052	269	532	795	54648	14°51′46″,2	9°53′1″,5	4°58′44″,7	160° 9′29″,1	69
244	197	188	291	42120	81. 49. 43,6	39.45.13,5	42. 4. 30,1	56. 5. 46,3	195
668	221	388	555	90288	50. 41. 32,5	32. 44.49,6	17.56.42,9	111. 21. 44,6	171
1244	317	628	939	70200	13. 41. 8,0	9. 6.15,6	4.34.52,4	161. 43. 59,6	75
484	123	244	365	9828	12. 40. 49,4	8. 26.17,5	4. 14. 31,9	163. 4. 38,7	27

c) 1. $\cos \alpha = \frac{d^2 + (a-c)^2 - b^2}{2d(a-c)}$, $\cos \beta = \frac{b^2 + (a-c)^2 - d^2}{2b(a-c)}$, $e^2 = \frac{(a^2 - b^2)c - (c^2 - d^2)a}{a - c}$, $f^2 = \frac{(a^2 - d^2)c - (c^2 - b^2)a}{a - c}$, etc.

2. $\sin(\gamma - \alpha) = \frac{a-c}{b} \sin \alpha$, $\beta = 180^0 - \gamma$, $d = \frac{b \sin \beta}{\sin \alpha}$, $F = \frac{1}{2}(a + c)b \sin \beta$.

3. $\sin \beta = \frac{h}{b}$, $\sin \alpha_2 = \frac{h}{e}$, $a = \sqrt{e^2 - h^2} + \sqrt{b^2 - h^2}$, $d^2 = b^2 + (a - c)^2 - 2(a - c)\sqrt{b^2 - h^2}$, $\sin \alpha = h : d$.

4. $\sin \beta_1 = \frac{e \sin \varphi}{a + c}$, $\alpha_2 = \varphi - \beta_1$, $f = \frac{a+c}{\sin \varphi} \sin \alpha_2$, $d^2 = e^2 + c^2 - 2ec \cos \alpha_2$, $b^2 = a^2 + e^2 - 2ae \cos \alpha_2$.

5. $\cos \beta_1 = \frac{f^2 + s^2 - e^2}{2fs}$, $\cos \alpha_2 = \frac{e^2 + s^2 - f^2}{2es}$, $\sin \beta = \frac{e}{b} \sin \alpha_2$, $a = \frac{b \sin(\beta + \alpha_2)}{\sin \alpha_2}$.

6. $a = \frac{2F}{h} - c$, $f = \frac{h}{\sin \beta_1}$, $\operatorname{tang} \alpha_2 = \frac{h^2}{2F - h^2 \operatorname{cotg} \beta_1}$, $e = \frac{h}{\sin \alpha_2}$, $b^2 = a^2 + e^2 - 2ae \cos \alpha_2$, $d^2 = a^2 + f^2 - 2af \cos \beta_1$.

7. $b = \frac{(a-c)\sin\alpha}{\sin(\alpha+\beta)}$, $d = \frac{(a-c)\sin\beta}{\sin(\alpha+\beta)}$,
$f^2 = a^2 + d^2 - 2ad\cos\alpha$.

8. $\alpha_2 = \varphi - \beta_1$, $e = \frac{(a+c)\sin\beta_1}{\sin\varphi}$, $f = \frac{(a+c)\sin\alpha_2}{\sin\varphi}$,
$b^2 = a^2 + e^2 - 2ae\cos\alpha_2$, $d^2 = a^2 + f^2 - 2af\cos\beta_1$.

9. $\cos\beta = \frac{a^2+b^2-e^2}{2ab}$, $\cos\alpha_2 = \frac{a^2+e^2-b^2}{2ae}$,
$\operatorname{tang}\frac{1}{2}\delta = \frac{e\sin\alpha_2}{s - e\cos\alpha_2}$, $d = \frac{e\sin\alpha_2}{\sin\delta}$, $c = \frac{e\sin(\delta+\alpha_2)}{\sin\delta}$.

10. $\sin\frac{1}{2}(\alpha+\beta) = \frac{a-c}{b-d}\sin\frac{1}{2}(\alpha-\beta)$,
$b + d = (a-c)\cdot\frac{\cos\frac{1}{2}(\alpha-\beta)}{\cos\frac{1}{2}(\alpha+\beta)}$, $\sin\beta_1 = \frac{d}{e}\sin\alpha$,
$a = \frac{e}{\sin\alpha}\sin(\alpha+\beta_1)$.

11. $k = s : 4\cos\frac{1}{2}\alpha \,.\, \cos\frac{1}{2}\beta \,.\, \sin\frac{1}{2}(\alpha+\beta)$, $b = k\sin\alpha$,
$d = k\sin\beta$, $a - c = k\sin(\alpha+\beta)$,
$\sin\beta_1 = d \,.\, \sin\alpha : e$, $a = e \,.\, \sin(\alpha+\beta_1) : \sin\alpha$.

12. $\varphi - \beta_1 = \alpha_2$, $c = \frac{s\cos\frac{1}{2}\varphi}{\cos\frac{1}{2}(\alpha_2-\beta_1)} - a$, $e = \frac{a\sin\beta_1}{\sin\varphi}$,
$f = \frac{a\sin\alpha_2}{\sin\varphi}$, $b^2 = a^2 + e^2 - 2ae\cos\alpha_2$.

13. $\beta = 180^0 - \gamma$, $b = \frac{h}{\sin\gamma}$, $d = \frac{h}{\sin\alpha}$,
$a - c = \frac{h\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$, $a + c = u - b - d$.

Zahlenbeispiele zu c) 1—13.

a	b	c	d	e	f	F	α	β
428	289	260	257	307,936	471	87720	82° 50′ 50″,4	61° 55′ 39″,1
268	255	0	281	281	255	30954	55. 17. 31,0	64. 56. 32,6
1004	223,395	696	305	943	807	175950	42. 44. 28,5	67. 54. 46,7
436	153	12	377	388,23	159	30240	20. 58. 58,6	61. 55. 39,1
1676	145	1144	425	1562,4	1263	122670	12. 48. 44,2	36. 52. 11,6

α_2	β_1
41° 7′ 50″,5	32° 46′ 44″,7
55. 17. 31,0	64. 56. 32,6
12. 40. 49,4	14. 51. 46,2
20. 20. 55,4	58. 6. 33,2
3. 11. 31,6	3. 56. 59,6

d) **1.** $\sin\gamma_1 = a : 2r$, $\sin\delta_1 = b : 2r$, $\sin\alpha_1 = c : 2r$,
$\delta = \delta_1 + \gamma_1$, $\alpha = \alpha_1 + \delta_1$, $\beta = 180^0 - \delta$,
$\gamma = 180^0 - \alpha$, $\beta_1 = \beta - \alpha_1$, $d = 2r\sin\beta_1$,
$e = 2r\sin\beta$, $f = 2r\sin\alpha$, $F = \frac{1}{2}(ab + cd)\sin\beta$.

2. $\sin\gamma_1 = a : 2r$, $\sin\delta_1 = b : 2r$, $\alpha_1 = \alpha - \delta_1$,
$c = 2r\sin\alpha_1$, $\gamma = 180^0 - \alpha$, $\beta_1 = \gamma - \gamma_1$,
$d = 2r\sin\beta_1$, $\beta = \alpha_1 + \beta_1$, etc.

3. $\sin\gamma_1 = a : 2r$, $\sin\alpha_1 = c : 2r$, $\delta_1 = \alpha - \alpha_1$,
$b = 2r\sin\delta_1$, $\gamma = 180^0 - \alpha$, $\beta_1 = \gamma - \gamma_1$,
$d = 2r\sin\beta_1$.

4. $\sin\gamma_1 = a : 2r$, $\sin\alpha_1 = c : 2r$, $2\beta_1 = \sigma - \alpha_1 - \gamma_1$,
$\beta = \alpha_1 + \beta_1$, $\gamma = \beta_1 + \gamma_1$, $\alpha = 180^0 - \gamma$,
$\delta = 180^0 - \beta$, $d = 2r\sin\beta_1$, $b = 2r\sin\delta_1$.

5. $\sin\alpha = f : 2r = \sin\gamma$, $\sin\beta = e : 2r = \sin\delta$,
$2\gamma_1 = \gamma + \delta - \varphi$.

6. $\delta = 180^0 - \beta$, $\gamma_1 + \delta_1 = \delta$,
$\tang\frac{1}{2}(\gamma_1 - \delta_1) = (a - b)\cotg\frac{1}{2}\beta : (a + b)$.

7. $\cos\beta = \dfrac{a^2 + b^2 - e^2}{2ab}$, $\cos\gamma_1 = \dfrac{b^2 + e^2 - a^2}{2be}$,
$\cos\delta_1 = \dfrac{a^2 + e^2 - b^2}{2ae}$, $\sin\alpha = \dfrac{f\sin\gamma_1}{a}$, $\alpha_1 = \alpha - \delta_1$,
$\beta_1 = \beta - \alpha_1$, $d = a\sin\beta_1 : \sin\gamma_1$.

8. $\delta = 180^0 - \beta$, $\sin\gamma_1 = a\sin\beta : e$, $\sin\alpha_1 = c\sin\beta : e$,
$\beta_1 = \beta - \alpha_1$, $\gamma = \beta_1 + \gamma_1$.

9. $f_2 = \dfrac{e_1 \cdot e_2}{f_1}$, $\cos\beta_1 = \dfrac{a^2 + f_1^2 - e_1^2}{af_1}$,
$\cos\delta_1 = \dfrac{a^2 + e_1^2 - f_1^2}{ae_1}$, $c = \dfrac{e_2\sin(\beta_1 + \delta_1)}{\sin\delta_1}$,
$b^2 = f_1^2 + e_2^2 - 2f_1e_2\cos(\beta_1 + \delta_1)$,
$d^2 = e_1^2 + f_2^2 - 2e_1f_2\cos(\beta_1 + \delta_1)$.

10. $CE = q = -\frac{1}{2}c + \sqrt{p(a + p) + \frac{1}{4}c^2}$,
$\cos\beta = \dfrac{q^2 - b^2 - p^2}{2bp}$, $\cos\gamma = \dfrac{p^2 - b^2 - q^2}{2bq}$,
$\alpha = 180^0 - \gamma$, $\delta = 180^0 - \beta$,
$d = (a + p)\sin(\alpha + \delta) : \sin\delta$.

11. $d = \dfrac{p^2 - ac}{b}$, $\cos\beta = \dfrac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}$,
$\cos\alpha = \dfrac{a^2 + d^2 - b^2 - c^2}{2(ad + bc)}$.

8. Seiten: $41^0\,48'\,32''$; $42^0\,50'\,1''$; $59^0\,10'\,12''$.
Winkel: $50^0\,43'\,42''$; $52^0\,8'\,23''$; $94^0\,18'\,56''$.

9. $\alpha = 109^0\,28'\,16''$; $\operatorname{tang} \frac{1}{2}\alpha = \sqrt{2}$; $\beta = 70^0\,31'\,44''$,
$O = 12\,a^2 \sin\alpha = 8\,a^2\sqrt{2}$, $V = \frac{16}{9}\,a^3\sqrt{3}$.

10. $\operatorname{tang} \frac{1}{2}\alpha = b : \sqrt{3b^2 - a^2}$, $\operatorname{tang} \frac{1}{2}\beta = 2\sqrt{3b^2 - a^2} : a$.

11. $\cos x = \dfrac{\sin\beta \sin\gamma - \cos\alpha}{\cos\beta\cos\gamma}$;
$$\operatorname{tang} \tfrac{1}{2}x = \sqrt{\frac{\sin\frac{1}{2}(\alpha+\beta-\gamma)\,\sin\frac{1}{2}(\alpha-\beta+\gamma)}{\cos\frac{1}{2}(\alpha+\beta+\gamma)\,\cos\frac{1}{2}(\beta+\gamma-\alpha)}}.$$

12. $\cos x = \sin\varphi\sin\varphi' + \cos\varphi\cos\varphi'\cos(l - l')$; $15\,x^0$.
a) 64,5 M., b) 617,5 M., c) $3\frac{3}{4}$ M.

13. $\cos(l - l') = \dfrac{\cos\frac{1}{15}d^0 - \sin\varphi\sin\varphi'}{\cos\varphi\cos\varphi'}$;
a) circa 44^m; b) circa $76^0\,22'$ w. L.

14. $75^0\,38'$ von Süden nach Osten.

15. $58^0\,6'\,15''$ n. Br., $4^0\,50'\,22''$ w. L.

16. Berechnung eines Dreiecks aus zwei Seiten $90^0 - b$, e und dem eingeschlossenen Winkel $90^0 - l$; $\delta = 32^0\,23'\,47'',5$, $\alpha = 301^0\,48'\,17''$.

17. Ebenso aus $90^0 - \delta$, e und $90^0 + \alpha$; $b = -5^0\,40'$, $l = 68^0\,29'\,19''$.

18. Berechnung eines Dreiecks aus zwei Seiten $90^0 - h$, $90^0 - \delta$ und einem gegenüberliegenden Winkel $180^0 - \alpha$; $w = 45^0\,42'$ ($134^0\,18'$), $\varphi = 67^0\,58'\,55''$.

19. Ebenso aus $90^0 - h$, $90^0 - \delta$ und w; im Zahlenbeispiel $\sin\varphi = \sin h : \sin\delta$; $\varphi = 54^0\,43'\,20''$.

20. Aus zwei Seiten $90^0 - \delta$, $90^0 - \varphi$ und dem eingeschlossenen Winkel w die dritte Seite $90^0 - h$ zu berechnen. $h = 65^0\,37'\,30''$.

21. Berechnung eines Winkels $180^0 - \alpha$ aus den drei Seiten p, z, $90^0 - \varphi$; $\alpha = 97^0\,53'\,51'',4$.

22. Gegeben eine Seite $90^0 - \varphi$ und die anliegenden Winkel $180^0 - \alpha$, w; gesucht eine Seite $90^0 - \delta$; $\delta = +20^0 48' 14'',3$.

23. Berechnung eines Dreiecks aus zwei Seiten p, $90^0 - \varphi$ und dem eingeschlossenen Winkel w; $h = 58^0\,25'\,15''$, $\alpha = 27^0\,32'$.

24. Berechnung eines Winkels w aus den drei Seiten $90^0 - h$, $90^0 - \varphi$, $90^0 - \delta$; $x = \dfrac{w'}{15} = 3^h\,59^m\,27^s$ (wahre Sonnen-Zeit).

25. $\cos w' = -\frac{\sin r + \sin\varphi \sin\delta}{\cos\varphi \cos\delta} = -\frac{\sin 35'}{\cos\varphi\cos\delta} - \operatorname{tang}\varphi \operatorname{tang}\delta$.

26. Für den wahren Sonnenaufgang ist $\cos w = -\operatorname{tang}\varphi \operatorname{tang}\delta$, daher $2 \sin\frac{1}{2}(w - w') \sin\frac{1}{2}(w + w_1) = -\frac{\sin r}{\cos\varphi\cos\delta}$;

$$w' - w = \frac{\sin r}{\cos\varphi\cos\delta\cos w}.$$

Der Zeitunterschied ist derselbe für je zwei Tage des Sommers und des Winters, welche gleiche nördliche und südliche Declination der Sonne haben, insbesondere also auch für den längsten und den kürzesten Tag; er ist am kleinsten für $\delta = 0$, d. h. zur Zeit der Tag- und Nachtgleichen, er wächst mit der nördlichen und südlichen Declination der Sonne und ist zur Zeit der Sonnenwenden am grössten.

27. $\cos w$ nach 25; $\frac{2w}{15}$ Stunden. Nimmt man an, die Declination der Sonne ändere sich gleichmässig während 24 Stunden und sei am wahren Mittag gleich δ, so ist näherungsweise

$$-\sin r = \sin\varphi \sin\left(\delta - \frac{nw}{360^0}\right) + \cos\varphi\cos\left(\delta - \frac{nw}{360^0}\right)\cos w',$$

$$-\sin r = \sin\varphi \sin\left(\delta + \frac{nw}{360^0}\right) + \cos\varphi\cos\left(\delta + \frac{nw}{360^0}\right)\cos w'',$$

wo w nach 25 zu bestimmen ist. Die gesuchte Zeit ist dann $(w' + w'') : 15$ Stunden.

28. $\varphi = 90^0 - (\delta + r)$, $\delta = 23^0 27' 40''$, $r = 35'$, $\varphi = 65^0 57' 20'$, (für den Sonnen-Mittelpunkt).

29. Unter der Breite $\varphi > 66^0\ 0'\ 20''$ bleibt der Sonnen-Mittelpunkt sichtbar von dem Momente, in welchem die Declination der Sonne $\delta = 90^0 - (\varphi + r)$ ist bis zu dem Momente, in welchem sie wieder die gleiche wird. Für den äussersten Sonnenrand ist $\delta = 90^0 - (\varphi + r + 16')$ zu setzen.

30. Sind a, a' die beobachteten Differenzen der Azimuthe in Beziehung auf den ersten Stern, h, h', h'' die beobachteten Höhen, so setze man

$$\operatorname{tang}\vartheta = \operatorname{tang}\tfrac{1}{2}(h' + h)\operatorname{tang}\tfrac{1}{2}(h' - h)\operatorname{cotg}\tfrac{1}{2}a,$$
$$\operatorname{tang}\vartheta' = \operatorname{tang}\tfrac{1}{2}(h'' + h)\operatorname{tang}\tfrac{1}{2}(h'' - h)\operatorname{cotg}\tfrac{1}{2}a',$$
$$\operatorname{tang}\psi = \frac{\operatorname{cotg}\frac{1}{2}(h'' - h)\sin\frac{1}{2}a'\cos\vartheta}{\operatorname{cotg}\frac{1}{2}(h' - h)\sin\frac{1}{2}a\cos\vartheta'},$$ und es ist

$$\operatorname{tang}[\alpha + \tfrac{1}{2}(\vartheta' + \vartheta) + \tfrac{1}{4}(a' + a)]$$
$$= \operatorname{tang}[\tfrac{1}{2}(\vartheta' - \vartheta) + \tfrac{1}{4}(a' - a)] \operatorname{tang}(45^0 + \psi).$$

Hieraus ergiebt sich das zwischen 0^0 und 180^0 liegende erste Azimuth α. Ferner ist

$$-\operatorname{tang}\varphi = \operatorname{cotg}\tfrac{1}{2}(h' - h)\sin\tfrac{1}{2}a\sin(\alpha + \varphi + \tfrac{1}{2}a) : \cos\varphi$$

und

$$\sin\delta = \sin\varphi\sin h - \cos\varphi\cos h\cos\alpha.$$

31. Bedeutung der Buchstaben, wie vorher.

$$\operatorname{tang}\psi = \frac{\cos\frac{1}{2}(\delta + \delta')\sin\frac{1}{2}(\delta - \delta')\sin\frac{1}{2}a'}{\cos\frac{1}{2}(\delta + \delta'')\sin\frac{1}{2}(\delta - \delta'')\sin\frac{1}{2}a}$$

$$\operatorname{tang}[\alpha + \tfrac{1}{4}(a + a')] = \operatorname{tang}\tfrac{1}{2}(a' - a)\operatorname{tang}(45^0 + \psi),$$

wo α zwischen 0 und 180^0 liegt. Ferner

$$\operatorname{tang}\vartheta^2 = \sin\tfrac{1}{2}\alpha^2\,\frac{2\cos\frac{1}{2}(\delta + \delta')\sin\frac{1}{2}(\delta - \delta')}{\sin(\alpha + \frac{1}{2}a)\sin\frac{1}{2}a\sin\delta},$$

$$\operatorname{tang}\eta^2 = \cos\tfrac{1}{2}\alpha^2\,\frac{2\cos\frac{1}{2}(\delta + \delta')\sin\frac{1}{2}(\delta - \delta')}{\sin(\alpha + \frac{1}{2}a)\sin\frac{1}{2}a\sin\delta},$$

$$\cos(\varphi + h) = -\sin\delta : \cos\vartheta^2,\quad \cos(\varphi - h) = \sin\delta\cos 2\eta : \cos\eta^2.$$

Schlusswort.

Die Resultate zu den nicht mit besonderen Zahlenbeispielen versehenen Aufgaben des §. 27 sind mit Rücksicht auf die beigegebene Anleitung zur Auflösung, um den Umfang des Resultatenheftes nicht ohne Noth zu steigern, weggelassen worden. Für die Zahlenbeispiele zu denselben ist die Tabelle, Seite 161 bis 164, beigegeben worden, welche in jedem einzelnen Fall auch die Resultate liefert. Einzelne kleine Unregelmässigkeiten der auf Ersparung von Raum berechneten Schreibweise der Resultate bittet der Verf. zu entschuldigen.

Auf Seite 38 des vorliegenden Heftes ist einzuschalten:

§. 25 b.

20. 1 : 1,87939. **21.** 5,438; 6,857. **22.** 0,1729; 0,1990. **23.** 18^0, 36^0, 126^0. **24.** 72^0, 45^0, 63^0; 1,4142; 1,0515; 1,3249; und auf Seite 48:

43. b) $\beta = 82^0\,59',2$, $\gamma = 70^0\,42',3$, $b = 11{,}197$, $c = 10{,}648$, $F = 26{,}421$.

RESULTATE

DER

RECHNUNGS-AUFGABEN IN DER SAMMLUNG

VON

AUFGABEN UND BEISPIELEN

AUS DER

TRIGONOMETRIE UND STEREOMETRIE

HERAUSGEGEBEN

VON

DR. FRIEDRICH REIDT,

OBERLEHRER AM GYMNASIUM UND DER HÖHEREN BÜRGERSCHULE IN HAMM.

II. THEIL: STEREOMETRIE.

ZWEITE AUFLAGE.

LEIPZIG,

DRUCK UND VERLAG VON B. G. TEUBNER.

1878.

Resultate der Rechnungs-Aufgaben aus der Stereometrie.

Einleitung.

1. 6, $\frac{n(n-1)}{1 \cdot 2}$. **2.** $\frac{n(n-1)-\alpha(\alpha-1)-\beta(\beta-1)-\gamma(\gamma-1)}{1 \cdot 2}+3$.

3. $\frac{n(n-1)p^2}{1 \cdot 2}+n$, 58. **4.** $\frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}$, 35.

5. ab, 45. **6.** $\frac{n(n-1)}{1 \cdot 2}$, 6.

7. $\frac{n(n-1)}{1 \cdot 2}$, 15. **8.** 4. **13.** $\frac{n(n-1)}{1 \cdot 2}$, 10.

14. $\frac{n(n-1)-\alpha(\alpha-1)-\beta(\beta-1)-\gamma(\gamma-1)}{1 \cdot 2}+2$; 49.

15. $\frac{p \cdot (p-1)}{1 \cdot 2}$ für $p=\frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}$; 24090.

Erster Abschnitt. I. Capitel.

§. 1.

32. $\sqrt{a^2-b^2}=\sqrt{(a-b)(a+b)}$; α) 10,4; β) 31,5.

33. $\sqrt{a^2+r^2}$; α) $32\frac{1}{8}$; β) $14\frac{1}{20}$.

34. $\sqrt{\frac{a^2n^2-b^2m^2}{n^2-m^2}}$; α) 21; β) 132.

35. $\sqrt{(a-b)^2+c^2}$; α) $1\frac{1}{60}$; β) 38,9.

36. $\sqrt{a^2+b^2-2c^2}$; α) 4; β) 9.

37. $F_1=\frac{3}{4}(b^2-c^2)\sqrt{3}$; $F_2=\frac{1}{4}\sqrt{3(b^2-c^2)(b^2+3c^2)}$.

38. $\frac{1}{2}a\sqrt{2}$; **39.** $(bm \pm an):(m \pm n)$.

40. $a \cdot \cos\alpha$; 221; 3,3. **41.** $\operatorname{tang}\alpha=r:p$, $\alpha=60^0\,27'$.

42. $\cos\alpha=a:b$, $\alpha=70^0$. **43.** $(a-b):\cos\alpha=2{,}24$.

§. 2.

22. $a\sqrt{2}$. **23.** $\frac{1}{2}a$.

24. $\sqrt{bc(b+c+a)(b+c-a)}:(b+c)=4\frac{16}{41}$.

25. $\sqrt{a^2+b^2}=2$. **26.** 60^0. **27.** $a\,.\cos\varphi=3992$.
28. $b:a=\cos\varphi,\ \varphi=39^0$. **29.** $a:\sin\varphi=73{,}09$.
30. $\operatorname{tang}\varphi=(b-c):a,\ \varphi=18^0$.
31. $\cos\varphi=\cos\alpha\,.\cos\beta,\ \varphi=45^0$.
32. $\cos\varphi=\cos\alpha:\cos\beta,\ \varphi=5^0\,40'$.
33. $\cos\varphi=\frac{a}{m}\sqrt{\frac{m^2-n^2}{a^2-b^2}}=\frac{1}{2}\sqrt{3},\ \varphi=30^0;$

$\cos\varphi_1=\frac{b}{n}\sqrt{\frac{m^2-n^2}{a^2-b^2}}=\frac{1}{2}\sqrt{2},\ \varphi_1=45^0.$

34. $\frac{2hm}{\sqrt{4m^2-n^2}},\ \frac{2hm^2}{n\sqrt{4m^2-n^2}};\ 34;\ 19\tfrac{4}{15}$.
35. $F=\sqrt{s\,.(s-a)\,.(s-b)\,.(s-c)}$,

$2s=a+b+c;\ \operatorname{tang}x=\frac{4hF}{abc},\ x=76^0\,23'\,21''.$

§. 3.

18. $\frac{p+q}{p}\,.\,a=4{,}95$. **19.** $\left(\frac{p+q}{p}\right)^2.\,F$.
20. $\sin\varphi=\frac{c\,.\sin\alpha}{d},\ \varphi=67^0$. **21.** $h\,.\sqrt{3}$.
22. $\cos\varphi=\frac{a^2+b^2-c^2+(d-e)^2}{2ab}$.

II. Capitel.

§. 4.

34. 30^0. **35.** $\sqrt{a^2-\frac{1}{4}b^2}=224$. **36.** $b\,.a':a=5{,}20$.
37. $\sqrt{a^2+b^2+c^2}=29$. **38.** $a\,.\operatorname{tang}\alpha=5{,}419$.
39. $\cos\varphi=\frac{b^2+c^2-a^2}{2bc},\ \varphi=60^0$.
40. $\sin\alpha:\sin\beta:\sin\varphi=8615:8000:9231$.
41. $F\,.\cos\varphi=717$. **42.** $F\,.\cos\varphi=12{,}31$.
43. $\sin\varphi=\sqrt{\sin\alpha^2+\sin\beta^2},\ \varphi=50^0$.
44. $\cos\varphi=\frac{l^2\sin\alpha\,.\sin\beta\pm\sqrt{(p-l\cos\beta)(p+l\cos\beta)(p-l\cos\alpha)(p+l\cos\alpha)}}{(l+p)\,(l-p)}$.
45. $h=\frac{d\,.\sin\varphi}{\sin\delta},\ s=\sqrt{h^2+\frac{1}{4}a^2},\ \operatorname{tang}\beta=\frac{2h}{a},$

$\operatorname{tang}\frac{1}{2}\alpha=\frac{a}{2h},\ F=\frac{1}{2}\,\frac{ad\sin\varphi}{\sin\delta};\ s=229;$

$\alpha=30^0\,22'\,43'',2;\ \beta=74^0\,48'\,38'',4;\ F=13260.$

46. $x = \sqrt{p^2 \sin \alpha^2 + q^2}$; $\cos \varphi = p : x$,
$\cos \psi = \frac{q}{x}$; $x = 17$, $\varphi = 22^0$, $\psi = 52^0$.

47. $\sin \varphi^2 = \frac{m^2(q^2 - p^2)}{n^2 q^2 - m^2 p^2}$, $\sin \psi^2 = \frac{n^2(q^2 - p^2)}{n^2 q^2 - m^2 p^2}$; 60^0, 45^0.

48. $\cos x = \frac{\cos \alpha - \sin \beta \sin \gamma}{\cos \beta \cos \gamma}$,
$\sin \varphi = \frac{1}{\sin \alpha} \cdot \sqrt{\sin \beta^2 + \sin \gamma^2 - 2 \sin \beta \sin \gamma \cos \alpha}$;
$x = 142^0 44'$, $\varphi = 36^0 17' 31'',4$.

§. 5.

33. $\sqrt{a^2 - b^2 + c^2} = 3,9$. **34.** $\sqrt{a^2 + b^2 + c^2} = 173$.

35. $\frac{1}{7} a \sqrt{7} = 2\frac{1}{2}$.

36. $\cos \varphi = (a^2 - b^2 + c^2 + d^2) : 2cd$; $\varphi = 52^0$.

III. Capitel.

§. 7.

29. $\sin x : \sin y = \sin \alpha : \sin \beta$.

30. $\cos \alpha = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}$; $\alpha = 18^0 37' 15''$;
$\beta = 26^0 59' 0''$; $\gamma = 140^0 14' 54'',5$.

31. $\cos a = \frac{\cos \alpha + \cos \beta \,.\, \cos \gamma}{\sin \beta \,.\, \sin \gamma}$; $a = 124^0 12' 32''$;
$b = 54^0 18' 14''$; $c = 97^0 12' 26''$.

32. $\cos \alpha = - \cos \beta \,.\, \cos \gamma + \sin \beta \,.\, \sin \gamma \,.\, \cos a$;
$\alpha = 101^0 44' 21''$; $b = 27^0 16' 9'',5$; $c = 88^0 11'$.
(Genauer: $\alpha = 101^0 44' 20''$; $b = 27^0 16' 8''$; $c = 88^0 12' 19''$.)

33. $\cos a = \cos b \,.\, \cos c + \sin b \,.\, \sin c \,.\, \cos \alpha$;
$a = 53^0 20' 10'',7$ $(19' 34'')$; $\beta = 54^0 52' 7'',7$ $(50'',4)$;
$\gamma = 32^0 25' 39'',4$ $(56'',6)$.

34. $\operatorname{tang} x = \operatorname{tang} \varphi : \sin \frac{m}{m+n} \alpha$;
$\operatorname{tang} y = \operatorname{tang} \varphi : \sin \frac{n}{m+n} \alpha$;
$x = 49^0 17' 26''$, $y = 42^0 20' 26''$.

35. Aus $\cos \gamma_1 = \operatorname{tang} \frac{1}{2} b : \operatorname{tang} x$, $\cos \gamma_2 = \operatorname{tang} \frac{1}{2} a : \operatorname{tang} x$, $\cos \gamma = \cos (\gamma_1 + \gamma_2)$ folgt $\operatorname{tang} x^2 = (\operatorname{tang} \frac{1}{2} a^2 + \operatorname{tang} \frac{1}{2} b^2 - 2 \operatorname{tang} \frac{1}{2} a \,.\, \operatorname{tang} \frac{1}{2} b \,.\, \cos \gamma) : \sin \gamma^2$, wo $\cos \gamma = (\cos c - \cos a \,.\, \cos b) : \sin a \,.\, \sin b$ ist.

36. $\cos\alpha = [2h^2(2t^2 - pq + qt + pt) - 3pqt^2] : \sqrt{[3q^2t^2 + 4h^2(q^2+t^2+qt)] \cdot [3p^2t^2 + 4h^2(t^2+p^2+pt)]}$ etc.

37. Vgl. 33.

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§. 8.

30. $\sqrt{a^2 + b^2 + c^2} = 125.$

31. $c\sqrt{a^2 + b^2} = 12\sqrt{106}$, $b\sqrt{a^2 + c^2} = 117$, $a\sqrt{b^2 + c^2} = 75.$

32. $\frac{1}{4}u + \frac{1}{2}\sqrt{\frac{1}{4}u^2 - 4f^2}$, $\frac{1}{4}u - \frac{1}{2}\sqrt{\frac{1}{4}u^2 - 4f^2}$, $\sqrt{d^2 - \frac{1}{4}u^2 + 2f}.$

33. $F\sqrt{2} = 10.$ **34.** $\frac{1}{16}u\sqrt{u^2 + 64p^2} = 15$; 0,65.

35. Er ist ein regelmässiges Sechseck und $1\frac{1}{2}$ mal so gross als jedes der Dreiecke.

36. a) $u = 3a\sqrt{2}$, $F = \frac{1}{2}a^2\sqrt{3}.$
b) Dieselben sind einander gleich.

37. Ein gleichschenkeliges Dreieck und ein symmetrisches Fünfeck. Die Flächeninhalte der Dreiecke sind bezüglich gleich $\frac{1}{16}a^2\sqrt{7}$ und $\frac{1}{16}a^2\sqrt{15}$. α) 1,1575; 1,6944. β) 2,5257; 3,6972.

38. Sie sind gleich gross.

39. Diagonalaxen: $a\sqrt{2}$ und $a\sqrt{6}$, spitzer Neigungswinkel der Flächen $\varphi = 70^0\,31'\,43'',3$, $\cos\varphi = \frac{1}{3}$, spitzer Neigungswinkel der Kanten gegen die Flächen $\psi = 54^0\,44'\,7'',1$, $\cos\psi = \sqrt{\frac{1}{3}}.$

40. $19^0.$

41. Die eine ist ein regelmässiges Sechseck mit der Seite $\frac{1}{3}a\sqrt{6}$; die andere entweder ein Sechseck, in dem je zwei gegenüberliegende Seiten gleich und parallel sind, oder ein Rechteck. Die Flächeninhalte verhalten sich wie 1 : $\cos\alpha$.

42. $\frac{1}{3}a^2 : \cos\alpha.$

43. $x^2 = b^2 + d^2 - 2bd\cos\beta$, $y^2 = a^2 + d^2 - 2ad\cos\alpha$, $16F^2 = 4c^2x^2 - (c^2 + x^2 - y^2)^2$; 1800.

44. Aus a, b, c berechne man den Flächenwinkel φ an der genannten Seitenkante, aus a, α und b, β zwei Grundkanten a_1, b_1, aus φ, α, β den Winkel γ zwischen a_1 und b_1 und den

Flächenwinkel δ an a_1, dann die dritte Grundkante c_1 aus a_1, b_1 und γ, endlich aus h, δ und α die Seitenkante.

§. 9.

44. $\sqrt{h^2 + \frac{1}{3}a^2} = 11{,}4.$ **45.** $\sqrt{2(b+h)(b-h)} = 0{,}4.$

46. $\sqrt{(b+a)(b-a)} = 3{,}08.$ **47.** $\frac{1}{2}a^2(\sqrt{2}+1) = 50.$

48. $h.\dfrac{\sqrt{\frac{1}{2}(G+g)} - \sqrt{g}}{\sqrt{G} - \sqrt{g}} = 7{,}265.$

49. $\dfrac{h.\sqrt[4]{g}}{\sqrt[4]{G} + \sqrt[4]{g}} = 1{,}2032.$

50. $\frac{1}{2}h^2(\sqrt{5}-1) = 0{,}61803\,h^2$ und $h^2(\sqrt{5}-2) = 0{,}23607\,h^2.$

51. $\frac{1}{x} = \frac{1}{a} + \frac{1}{h}$, $x = 2{,}4.$ **52.** $\frac{1}{x} = \frac{1}{h} + \frac{\sqrt{2}}{a}$, $x = 2.$

53. $n : \sqrt{12(4-n^2)} = 1 : 6.$ **54.** $\frac{1}{3}\sqrt{2}.$

55. $\dfrac{nr}{\sqrt{4-n^2}} = 8.$ **56.** $\dfrac{ab}{a+b} = \alpha)$ 1,875; $\beta)$ 1,2.

57. $\frac{1}{9}(G + 4\sqrt{Gg} + 4g)$ und $\frac{1}{9}(4G + 4\sqrt{Gg} + g)$; 19,6 und 25,6.

58. $q(m-1) : p = 4 : 17.$

59. $\cos\varphi = \frac{1}{3}a\sqrt{3} : b$, $\cos\psi = \frac{1}{3}a\sqrt{3} : \sqrt{4b^2 - a^2}$; $\varphi = 25^0\,46'\,30''$, $\psi = 44^0$.

60. a) $\sin\frac{1}{2}\varphi = \dfrac{\sqrt{9n^2+3}}{6n}$, $\cos\frac{1}{2}\varphi = \dfrac{\sqrt{27n^2-3}}{6n} = \frac{3}{4}$, $\varphi = 82^0\,49'\,10''$. b) $\cos\psi = 1 : 3n = \frac{1}{2}$, $\psi = 60^0$.

61. Neigungswinkel: $\operatorname{tang}\alpha_1 = \operatorname{tang}\alpha_2 = 3b : a$; $\operatorname{tang}\alpha_3 = 3b\sqrt{2} : a$; ebene Winkel der Seitenkanten gegen die Hypotenuse: $\operatorname{tang}\beta = \sqrt{18b^2 + a^2} : 3a$, gegen die Katheten an dem rechten Winkel $\operatorname{tang}\gamma = \sqrt{9b^2 + a^2} : a$, an den anderen Eckpunkten $\operatorname{tang}\delta = \sqrt{9b^2 + a^2} : 2a$. a) $\alpha_1 = 67^0\,22'\,48'',3$, $\alpha_3 = 73^0\,35'\,0'',0$, $\beta = 49^0\,42'\,25'',6$, $\gamma = 68^0\,57'\,44'',1$, $\delta = 52^0\,25'\,53'',0$. b) $\alpha_1 = 73^0\,44'\,23'',1$, $\alpha_3 = 78^0\,20'\,48'',3$, $\beta = 58^0\,47'\,8'',3$, $\gamma = 74^0\,21'\,27'',5$, $\delta = 60^0\,45'\,4'',0$.

62. tang $\alpha = p\sqrt{m^2 + n^2} : mn$, tang $\beta = n\sqrt{p^2 + m^2} : pm$, tang $\gamma = m\sqrt{p^2 + n^2} : pn$; $\alpha = 67^0\ 24'\ 41''$, $\beta = 59^0\ 11'\ 34''$, $\gamma = 39^0\ 48'\ 21''$.

63. $\sin \frac{1}{2}\varphi = \sin\left(\frac{n-2}{n} \cdot 90^0\right) : \cos \frac{1}{2}\alpha$; $\varphi = 165^0$.

64. $g = s \cdot \cos\alpha$, $G = 2s \cdot \cos\alpha$; $g = 254$, $G = 508$.

65. $\cos\alpha = \frac{\sqrt{m} - \sqrt{n}}{2\sqrt{m} \cdot \sin 36^0}$, $\alpha = 57^0\ 40'$.

66. $143^0\ 7'\ 48'',7$.

67. $x = a \cdot \sin\beta : \sin(\alpha + \beta)$, $y = a \cdot \sin\alpha : \sin(\alpha + \beta)$ $= b \cdot \sin\delta : \sin(\gamma + \delta)$, $z = b \cdot \sin\gamma : \sin(\gamma + \delta)$. Die Winkel ergeben sich aus den Seiten x, z, c des Dreiecks.

68. Es seien a, d und c, f die Längen der halbirten, b, e die der nicht halbirten Kantenpaare, so ist $\cos\varphi = (c^2 + f^2 - a^2 - d^2) : 2be$.

§. 10.

41. $rh\sqrt{2} = 1{,}4$. **42.** $\frac{1}{4}r\sqrt{12h^2 + 3r^2} = 2$.

43. $\frac{3}{4}d(4 - \sqrt{2})$. **44.** Wie $2 : \pi$. **45.** Wie $4 : \pi$.

46. $\frac{h}{c} \cdot \sqrt{(R+r+c)(R+r-c)(R-r+c)(-R+r+c)} = 1760$.

47. $h = \frac{1}{2}(\sqrt{d^2 + 2F} - \sqrt{d^2 - 2F})$,
$r = \sqrt{c^2 + \frac{1}{8}(d^2 + \sqrt{d^4 - 4F^2})}$;
$h_1 = 8$, $r = \frac{1}{2}\sqrt{261} = 8{,}0778$; $h_2 = 15$, $r_2 = 5$.

48. $\frac{1}{2}F\sqrt{3}$.

49. Berechne: $2s = a + b + c = 336$,

$\varrho = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} = 32$; $F = \varrho \cdot s = 5376$,

$R = \frac{abc}{4F} = 65$, $H = \sqrt{(d - R)(d + R)} = 72$,

$r = \frac{H^2\varrho \pm \varrho\sqrt{4\varrho^2 f^2 + f^2H^2 - 4\varrho^2H^2}}{4\varrho^2 + H^2}$,

$r_1 = 23\frac{109}{145}$, $r_2 = 12$ und $h = \frac{H(\varrho - r)}{\varrho}$,
$h_1 = 18\frac{81}{145}$, $h_2 = 45$.

50. $2\dot{a}r \sin\alpha = 5{,}723$.

51. $2ar\sqrt{1-\cos\alpha^2\cos\beta^2} = 119{,}405.$

52. $r = \frac{1}{2}\sqrt{\{a^2+b^2-2\sqrt{(ab+F)(ab-F)}\}} = 17{,}2616;$
$h = F:2r = 17{,}8432.$

§. 11.

24. $m^2r^2\pi:(m+n)^2.$

25. $\frac{h}{R-r}\{R-\sqrt{\frac{1}{2}(R^2+r^2)}\} = 6.$

26. $r\pi.$ **27.** $\sqrt{h^2+r^2}$; α) 2,29; β) 28,9; γ) 25,7.

28. $\sqrt{\frac{1}{2}(S^2+s^2)-a^2} = 17{,}6786.$

29. $a = \sqrt{\frac{1}{2}(S^2+s^2)-r^2}$; $s = \sqrt{2(a^2+r^2)-S^2}$;
$S = \sqrt{2(a^2+r^2)-s^2}.$

30. $\frac{1}{2}r\sqrt{2h^2+r^2} = 38\frac{1}{2}.$ **31.** $r\sqrt{3(r^2+h^2)}.$

32. $\frac{Rr}{R+r}.$ **33.** $\sqrt{[s^2-\frac{1}{4}a^2+(1-\frac{1}{2}\sqrt{2})ab]} = 6{,}705(1).$

34. $r = h \operatorname{tang}\frac{1}{2}\alpha = 7$; $s = h:\cos\frac{1}{2}\alpha = 25.$

35. $h = a.\sin\varphi = 308$; $S = \sqrt{a^2+r^2+2ar\cos\varphi} = 533$;
$s = \sqrt{a^2+r^2-2ar\cos\varphi} = 317.$

36. $\sin\varphi = h:a$; $\varphi = 69^0\,37'\,28''6$;
$s = \sqrt{a^2+r^2-2ar.\cos\varphi} = 42{,}1$;
$S = \sqrt{a^2+r^2+2ar\cos\varphi} = 54{,}1$;
$\sin\alpha = h:S$; $\alpha = 50^0\,55'\,36'',1.$

37. $h = S.\sin\alpha = 10$; $\sin\beta = S.\sin\alpha:s$;
$\beta_1 = 57^0\,25'\,0''$, $\beta_2 = 122^0\,35'\,0''$;
$\gamma_1 = 78^0\,40'\,55'',7$, $\gamma_2 = 13^0\,30'\,55'',7$;
$r = \frac{1}{2}s.\sin\gamma:\sin\alpha$; $r_1 = 8{,}39117$, $r_2 = 2$;
(1,99995); $\operatorname{tang}\varphi = h:(S.\cos\alpha - r)$;
$\varphi_1 = 78^0\,41'\,28'',2$, $\varphi_2 = 50^0\,0'\,0''.$

38. $\sin\beta = r.\sin\alpha:a$; $\beta = 36^0\,26'\,28'',6.$
$\varphi = \alpha+\beta$; $\varphi = 71^0\,55'\,50'',2.$
$S = a.\sin\varphi:\sin\alpha$; $S = 6{,}89.$
$s = \sqrt{a^2+r^2-2ar.\cos\varphi}$; $s = 5.$
$h = a.\sin\varphi$; $h = 4.$

§. 12.

77. $\varrho = \sqrt{r^2 - p^2}$; α) 2,99; β) 7,56; γ) 79,2.

78. $r = \sqrt{(m^2 \varrho_1^2 - n^2 \varrho^2) : (m^2 - n^2)} = 25$.

79. $r = \frac{1}{24} a \sqrt{219} = 219$. **80.** $R = \frac{1}{2} \sqrt{a^2 + 2r^2} = 28{,}5$.

81. $\varrho = \frac{3}{5} r = 15$; $p = \frac{4}{5} r = 20$.

82. $p = 3r \cdot \frac{m \pm \sqrt{9 - 8m}}{m + 9} = \frac{3}{5} r$. (Der zweite Werth für $m = 1$ ist $p = 0$.) Radius für die Halbkugel: $\frac{r}{m+9} \cdot \sqrt{-8m^2 + 90m \mp 18m\sqrt{9 - 8m}} = \frac{4}{5} r$. Radius für den Kegel: $\frac{r}{m+9} (9 \mp \sqrt{9 - 8m}) = \frac{4}{5} r$.

83. $r_1 = \frac{ma}{m+n} \cdot \frac{3 - \sqrt{3}}{2} = \frac{1}{3} a$;

$r_2 = \frac{na}{m+n} \cdot \frac{3 - \sqrt{3}}{2} = \frac{7 - 3\sqrt{3}}{6} a$.

84. Die Seitenlinie muss gleich der Summe der beiden Radien sein. $R = \sqrt{r \, . \, r_1} = 6{,}6$.

85. $G = 4a^2$; $r = \frac{a}{3}\sqrt{6}$; 2 : 1.

86. $r_1 = r(\sqrt{2} - 1)$; $r_2 = r(\sqrt{2} + 1)$. **87.** 7 : 3.

88. $p = \frac{hr^2(n-1)}{nr^2 + h^2}$ (oder $p = h$); Beisp. $p = \frac{hr^2}{2r^2 + h^2}$.

89. $f_1 + f_2 + f_3 = 2s = 471{,}38$;

$h^2 = \sqrt{s \, . \, (s - f_1)(s - f_2)(s - f_3)} : 4g$; $h = 1{,}85$;

$r = abc : 4g$; $R = \sqrt{r^2 + \frac{1}{4} h^2} = 434{,}4 \ldots$

90. 2 : 1. **91.** $\sqrt{5 + 2\sqrt{2}} : \sqrt{17} = 1 : \sqrt{5 - 2\sqrt{2}}$.

92. $\frac{R^2}{4(r - \varrho)} - \varrho = 4$. **93.** $x = r \, . \, \frac{\sqrt{h^2 + r^2} - r}{h + \sqrt{2}(\sqrt{h^2 + r^2} - r)}$.

94. R. **95.** $s : h$. **96.** $\frac{1}{2} R (3\sqrt{2} - 4)$.

97. $a_1 = 2r \, . \, \sin \frac{1}{2} \alpha = 37{,}0$; $b_1 = 2r \, . \, \sin \frac{1}{2} \beta = 143{,}2$; $c_1 = 2r \, . \, \sin \frac{1}{2} \gamma = 108{,}6$; $a_1 + b_1 + c_1 = 2s = 288{,}8$; $F = \sqrt{s(s - a)(s - b)(s - c)} = 816{,}24$.

98. $R^2 = r^2 + (a^2 - r^2)^2 : 4a^2 \sin \varphi^2$; $R = 25$.

99. Ist $m = \sqrt{R^2 - r^2} \, . \, \sin \varphi$, so ist $a = \sqrt{r^2 + m^2} \pm m$, $\operatorname{tang} \alpha = r : \sqrt{R^2 - r^2}$; $a_1 = 59{,}7299$, $\alpha_1 = 19^0 43' 54''$.

100. $(m^2 \operatorname{cotg} \frac{1}{2} \varphi^2 - n^2) : mn (\operatorname{cotg} \frac{1}{2} \varphi^2 - 1)$.

§. 13.

36. Tetraëder: $R = \frac{1}{4} a \sqrt{6}$, $r = \frac{1}{12} a \sqrt{6}$, $d = \frac{1}{4} a \sqrt{2}$.
Oktaëder: $R = \frac{1}{2} a \sqrt{2}$, $r = \frac{1}{6} a \sqrt{6}$, $d = \frac{1}{2} a$.
Ikosaëder: $R = \frac{1}{4} a \sqrt{10 + 2\sqrt{5}}$, $r = \frac{1}{12} a \sqrt{3}\,(3 + \sqrt{5})$, $d = \frac{1}{4} a \sqrt{6 + 2\sqrt{5}}$.
Hexaëder: $R = \frac{1}{2} a \sqrt{3}$, $r = \frac{1}{2} a$, $d = \frac{1}{2} a \sqrt{2}$.
Dodekaëder: $R = \frac{1}{4} a \sqrt{18 + 6\sqrt{5}}$, $r = \frac{1}{20} a \sqrt{250 + 110\sqrt{5}}$, $d = \frac{1}{2} a \sqrt{\frac{1}{2}(7 + 3\sqrt{5})}$.

37. 1 : 1. **38.** 1 : 1.

39. Sie ist ein regelmässiges Sechseck und gleich $1\frac{1}{2} F$.

40. Der Umfang ist stets gleich $3a$, der Inhalt ist gleich $\frac{3}{8}\sqrt{3}\,(a^2 - 2d^2)$.

41. Er ist constant gleich der doppelten Kante des Tetraëders.

42. Sie sind einander gleich.

43. $\sqrt{m^2 + \frac{4}{3} mn - \frac{4}{3} m \sqrt{mn}} : n = 1 : 3$.

44. $2\sqrt{6} : 3$. **45.** $(2 - \sqrt{2}) : 1$. **46.** $\sqrt{5 + 2\sqrt{5}} : \sqrt{3}$.

47. Die Umfänge verhalten sich wie $\frac{1}{4}(\sqrt{5} + 1) : 1 : 1$, oder der Umfang des ersten Schnitts verhält sich zum Umfang des zweiten, wie die Seite des dritten zur Kante des Körpers, und der zweite und dritte Schnitt haben gleiche Umfänge. Es ist $u_1 = \frac{5}{4} a (3 + \sqrt{5})$, $u_2 = u_3 = \frac{5}{2} a (\sqrt{5} + 1)$. Die Flächeninhalte sind $F_1 = \frac{1}{32} a^2 (7 + 3\sqrt{5})$, $F_2 = \frac{1}{8} a^2 \sqrt{650 + 290\sqrt{5}}$, $F_3 = \frac{5}{16} a^2 \sqrt{130 + 58\sqrt{5}}$, oder es ist $F_1 : F_2 = \sqrt{\frac{1}{2}(5 + \sqrt{5})} : 20$, $F_3 : F_2 = 5\sqrt{5} : 2$.

48. a) $\cos\alpha = \frac{1}{3}$, $\alpha = 70^0\,31'\,43'',6$; b) $\cos\frac{1}{2}\alpha = \frac{1}{3}\sqrt{3}$, $\alpha = 109^0\,28'\,16'',4$; c) $\sin\frac{1}{2}\alpha = \frac{2}{3}\sqrt{3} \,.\, \sin 54^0$, $\alpha = 138^0\,11'\,23''$; d) $\cos\alpha = -\sqrt{\frac{1}{5}}$, $\alpha = 116^0\,33'\,54'',2$.

49. $\sin\varphi = \frac{1}{3}\sqrt{3}$, $\varphi = 35^0\,15'\,52''$.

50. $H : h = 3{,}4549 : 1$ oder nahezu wie $919 : 266$.

51. $54^0\,44',1$; $70^0\,31',8$; $\frac{1}{4} a^2 \sqrt{2}$.

52. $\operatorname{tang}\varphi = 2m\sqrt{2} : (m + 3n) = \frac{2}{7}\sqrt{2}$ oder $\frac{4}{5}\sqrt{2}$. $\varphi = 22^0\,0'$ oder $48^0\,31'\,39''$. (Genauer $22^0\,0'\,6''$ und $48^0\,31'\,37''$.)

53. $h = \frac{1}{2}a$, $s = \frac{1}{3}a\sqrt{6}$, $\operatorname{tang}\frac{1}{2}\alpha = \frac{1}{2}\sqrt{2}$,
$\frac{1}{2}\alpha = 35^0\,15'\,52''$.

54. $h = \frac{1}{12}a\sqrt{6}$, $s = \frac{1}{4}a\sqrt{6}$, $\sin\frac{1}{2}\alpha = \frac{1}{3}\sqrt{6}$.

55. $0{,}23278\,a$.

§. 15.

1. $2a(a+2b)$. $\alpha)$ 2880,5; $\beta)$ $61\frac{1}{8}$; $\gamma)$ 0,0464.

2. $3a^2\sqrt{3}+6ab$. $\alpha)$ 12,4151; $\beta)$ 547,2836.

3. $4a\,.\left\{a(\sqrt{2}+1)+2b\right\} = 4a\,.\,(2{,}41421\,a+2b)$;
$\alpha)$ 300,35029; $\beta)$ 0,56027.

4. $\sqrt{2d^2-\frac{1}{4}u^2} = w = \pm 1{,}5$;
$O = \frac{1}{4}(\frac{1}{2}u-w)^2 + \frac{1}{2}\sqrt{2}\,(\frac{1}{4}u^2-w^2)$;
$O_1 = 20{,}25 + 54\sqrt{2} = 96{,}618$;
$O_2 = 36 + 54\sqrt{2} = 112{,}368$.

5. $\frac{2d^2}{a^2+b^2}\,ab + ac) + bc) = 846$.

6. $4ah + a^2(\sqrt{2}-1) = 215 + 25\sqrt{2} = 250{,}355$.

7. $3ab$. **8.** $2\,.\,240 + 90\,.\,50 = 4980$.

9. $a^2 = \frac{1}{5}O : (2+\sqrt{5+2\sqrt{5}})$. **10.** $a^2 + 2ab\sqrt{2}$.

11. $3a\,(\frac{1}{2}a\sqrt{3} + \sqrt{h^2+\frac{3}{4}a^2}) = 105{,}62$.

12. $a^2 = O^2 : (2O + 4h^2)$; $a = 20$.

13. $a^2 = \frac{1}{6}O\sqrt{3}$; $a = 8{,}77$.

14. $\sqrt{f} = 75$; $a = \frac{h}{p}\sqrt{f} = 112{,}5$; $h_1 = \sqrt{h^2+\frac{1}{4}a^2} = 237{,}75$;
$h_2 = \sqrt{p^2+\frac{1}{4}f} = 158{,}5$;
$O = f + a^2 + 2(a+\sqrt{f})\,(h_1-h_2) = 48000$.

15. $\frac{1}{16}u^2 + \frac{1}{2}\sqrt{32F^2+\frac{1}{64}u^4} = \frac{1}{16}\,.\,64 + \frac{1}{2}\,.\,72 = 40$.

16. $\frac{3}{4}\,[\sqrt{3}\,(s^2-h^2) + \sqrt{3(s^2-h^2)\,(s^2+3h^2)}\,] = 90{,}796$.

17. $g^2 + 2g\sqrt{h^2+\frac{1}{4}g^2} = 9{,}61 + 6{,}2\,.\,4{,}93 = 40{,}176$.

18. $\frac{5}{4}\sqrt{[(5-3\sqrt{5})\,l^2p^2 - \frac{5}{8}(3-\sqrt{5})\,p^4 + 2\,(5+\sqrt{5})\,l^4]}$
$= 5\sqrt{210-6\sqrt{5}} = 70{,}104$.

19. $\frac{1}{2}(ap_1+bp_2+cp_3) + \frac{1}{2}a\sqrt{h^2+p_1^2} + \frac{1}{2}b\sqrt{h^2+p_2^2}$
$+ \frac{1}{2}c\sqrt{h^2+p_3^2}$. Bedingungsgleichung:

$\frac{1}{2}(ap_1 + bp_2 + cp_3) = \sqrt{s(s-a)(s-b)(s-c)}$, für $2s = a + b + c$.

20. $\frac{a^2}{4}(3 + \sqrt{3}) = 2$.

21. $\frac{1}{4}b^2(3 + \sqrt{3}) - \frac{1}{4}a^2(3 - \sqrt{3}) = 100$.

22. Durch die Formel $F = \sqrt{s(s-a)(s-b)(s-c)}$ leicht zu lösen. Im besonderen Fall sind die vier Dreiecke congruent. $O = 4 \,.\, 240 = 960$.

23. a) $a^2\sqrt{3}$; b) $2a^2\sqrt{3}$; c) $5a^2\sqrt{3}$; d) $3a^2\sqrt{25 + 10\sqrt{5}}$.

24. $\frac{1}{2}a\sqrt{2}$. **25.** $\sqrt{3} : 1$. **26.** $1 : 2$. **27.** $\frac{1}{6}O\sqrt{3}$.

§. 16.

1. $M = 2r\pi h$; α) 2186,8; β) 24,85.

2. $h = M : 2r\pi$; α) 39,91; β) 1,622.

3. $r = M : 2h\pi$; α) 1,96; β) 200.

4. $M = 2r^2\pi = 802{,}3$.

5. $h = O : 2r\pi - r$, α) 10; β) 30.

6. $r = \sqrt{(O - M) : 2\pi} = 7$.

7. $r = \sqrt{\frac{O}{2\pi} + \frac{h^2}{4}} - \frac{h}{2}$; α) 80; β) 400.

8. $h = M : \sqrt{2\pi(O - M)} = 5$.

9. $M = a^2\pi\sqrt{2}$. **10.** $M = a^2\pi$.

11. Sein Radius ist das geometrische Mittel zwischen der Höhe und dem Durchmesser des Cylinders.

12. r. **13.** $(M_1 + M_2 + M_3)\sqrt{3} : a\pi$.

14. $\sqrt{r\pi(h + r) + h^2} - h = 12{,}8$. **15.** $2r\pi \,.\, a$.

16. $\frac{1}{2}\sqrt{2 - \sqrt{2}}\left[\sqrt{d^2 + \frac{2M}{\pi}} \pm \sqrt{d^2 - \frac{2M}{\pi}}\right]$, insbesondere:

$\frac{1}{2}d\sqrt{2 - \sqrt{2}} \,.\, (\sqrt{\tfrac{3}{2}} \pm \sqrt{\tfrac{1}{2}}) = \frac{1}{2}d\sqrt{(2 - \sqrt{2})(2 + \sqrt{3})}$.

17. $2\pi r^2 \,.\, \frac{3n^2 + 2mn - m^2}{n^2} = \frac{70}{9}r^2\pi = 62{,}39$.

18. $\frac{1}{2}F\pi = 100$.

19. $\frac{1}{3}r\pi(r + 2h) - \frac{1}{4}r^2\sqrt{3}$; $\frac{2}{3}r\pi(r + 2h) + \frac{1}{4}r^2\sqrt{3}$.

20. $\frac{1}{5}r\pi(r + h) + 2rh$. **21.** $\alpha : (360^0 - \alpha) = 2 : 7$.

22. Setze $a+b+c+d=2s$; $\sqrt{(s-a)(s-b)(s-c)(s-d)}=F$, $\sqrt{(ac+bd)(ad+bc)(ab+cd)}:4F=r$, so ist $O=2r\pi h+r^2\pi-2F\cdot\frac{r_1^2}{r^2}+2s\cdot\frac{r_1}{r}\cdot h$.

§. 17.

1. $RS\pi$; α) 788,1; β) 320.
2. $R\pi\sqrt{H^2+R^2}=10$.
3. $S\pi\sqrt{S^2-H^2}=15S\pi=5325$.
4. $R\pi(R+S)=411{,}8$.
5. $R\pi(R+\sqrt{R^2+H^2})=1999{,}91$.
6. $(S\sqrt{S^2-H^2}+S^2-H^2)\pi=500$. 7. $M:S\pi=75$.
8. $M:R\pi=36{,}5$. 9. $\frac{1}{R\pi}\sqrt{M^2-R^4\pi^2}=22{,}1$.
10. $\sqrt{-\frac{1}{2}H^2+\sqrt{\frac{1}{4}H^4+\frac{M^2}{\pi^2}}}=12{,}003$.
11. $\sqrt{\frac{1}{2}H^2+\sqrt{\frac{1}{4}H^4+\frac{M^2}{\pi^2}}}=289$.
12. $\frac{1}{2}\left(\sqrt{\frac{4O}{\pi}+S^2}-S\right)=29$. 13. $\frac{O}{R\pi}-R=17{,}6$.
14. $\sqrt{\frac{O}{\pi}\left(\frac{O}{R\pi}-2\right)}=2{,}47$.
15. $\sqrt{\left[\frac{1}{2}S^2-\frac{O}{\pi}+S\sqrt{\frac{1}{4}S^2+\frac{O}{\pi}}\right]}=2{,}31$.
16. $\frac{O}{\pi}:\sqrt{\frac{2O}{\pi}+H^2}=6{,}25014$.
17. $\left(\frac{O}{\pi}+H^2\right):\sqrt{H^2+\frac{2O}{\pi}}=47{,}241$.
18. $R=\sqrt{\frac{O-M}{\pi}}=3{,}8$; $S=\frac{M}{\sqrt{(O-M)\pi}}=4{,}61\,(4{,}6099)$; $H=\sqrt{S^2-R^2}=2{,}61$.
19. $\frac{H^2s\pi}{h^2}\sqrt{s^2-h^2}=1400$. 20. $\frac{R^2s\pi}{R-r}=600\,(599{,}99)$.
21. $\frac{R^2s\pi}{\sqrt{s^2-h^2}}=801{,}68$. 22. $\frac{R^2\pi}{R-r}\sqrt{(R-r)^2+h^2}=420$.
23. $\frac{H^2r\pi}{(H-h)^2}\sqrt{(H-h)^2+r^2}=710$.

24. $\frac{(r+\sqrt{s^2-h^2})^2}{\sqrt{s^2-h^2}} \cdot h = 400.$ 25. $\frac{S^2 r\pi}{S-s} = 615.$

26. $\frac{S^2\pi}{s}\sqrt{s^2-h^2} = 304\ (303{,}99).$

27. $\frac{h}{s}\sqrt{\frac{Ms}{\pi\sqrt{s^2-h^2}}} = 13{,}11.$

28. $sH : \sqrt{\frac{1}{2}H^2 + \sqrt{\frac{1}{4}H^4 + \frac{M^2}{\pi^2}}} = 8{,}5.$

29. $\frac{h}{H} \cdot \sqrt{\frac{1}{2}H^2 + \sqrt{\frac{1}{4}H^4 + \frac{M^2}{\pi^2}}} = 303.$

30. $M(R-r) : R^2\pi = 1045{,}5.$

31. $R(M - Rs\pi) : M = 8{,}349.$

32. $\frac{M \pm \sqrt{M^2 - 4Mrs\pi}}{2s\pi} = 44$ oder $11.$

33. $\sqrt{\frac{M\sqrt{s^2-h^2}}{s\pi}} = 119{,}1.$

34. $Mh : \sqrt{M^2 - R^4\pi^2} = 462\frac{8}{11}.$

35. $s\sqrt{M^2 - R^4\pi^2} : M = 666.$

36. $\frac{R-r}{R} \cdot \sqrt{\frac{M^2}{R^2\pi^2} - R^2} = \frac{316{,}8}{396} \cdot \sqrt{404^2 - 396^2}$
$= \frac{316{,}8}{396} \cdot 80 = 64.$

37. $S = M : R\pi = 261{,}2;\ H = \sqrt{S^2 - R^2} = 126;$
$R\,.\,(H - h) : H = 114{,}4.$

38. $R = \sqrt{\left[-\frac{1}{2}H^2 + \sqrt{\frac{1}{4}H^4 + \frac{M^2}{\pi^2}}\right]} = 43{,}68;$
$H(R - r) : R = 2{,}7133\ldots$

39. R wie vorher $= 4{,}6;\ R(H - h) : H = 4\frac{2}{11}.$

40. $R^2 = M\sqrt{s^2 - h^2} : s\pi;$
$R - \sqrt{s^2 - h^2} = 250{,}8 - 83{,}6 = 167{,}2.$

41. $R = \frac{M \pm \sqrt{M^2 - 4Mrs\pi}}{2s\pi} = 40{,}48;\ (404{,}8);$
$\sqrt{s^2 - (R - r)^2} = 10{,}26.$

42. $\frac{S-s}{S} \cdot \frac{M}{S\pi} = \frac{S-s}{S} \cdot 3 = 1.$

43. $R = M : S\pi = 8;\ s = S\,.\,(R - r) : R = 19{,}23.$

44. R (wie 41) $= 9$ oder $4{,}5$; $S = s \,.\, R : (R - r) = 4{,}9075$ oder $9{,}915$.

45. $S^2 = Ms : \pi\sqrt{s^2 - h^2} = Ms : 6\pi$; $S = 17{,}224$.

46. $R = M : S\pi = 21$; $h = s \,.\, \sqrt{S^2 - R^2} : S = 16{,}835s : S = 12{,}025$.

47. $R = M : S\pi = 14$; $s = S \,.\, h : \sqrt{S^2 - R^2} = S \,.\, h : 8{,}58 = 8{,}21$.

48. $O_1 = \pi(R^2 + r^2 + Rs + rs)$; $R = rS : (S - s) = 7{,}9(002)$; $O_1 = 500(,02)$.

49. $R = \sqrt{s^2 - h^2} + r = 8{,}0025$; $O_1 = 697{,}1$.

50. $r = \sqrt{s^2 - h^2} = 110$; $O_1 = 200000$.

51. $s = S \,.\, (R - r) : R = 201{,}686 \,.\, 2{,}3 : 35$; $O_1 = 2000$.

52. $R = Hr : (H - h) = 27{,}0006$; $s^2 = h^2 + (R - r)^2$; $O_1 = 1000$.

53. $\frac{O_1}{\pi} \cdot (S - s)^2 = r^2 [S^2 + (S - s)^2] + rs\,(S - s)(S + S - s)$; $\frac{O_1}{\pi} = 1{,}13$; $r = 0{,}5$.

54. $R^2 + Rs = \frac{O_1}{\pi} - r(r + s)$; $S = \frac{sR}{R - r} = 7{,}8$.

55. $s^3 - 13s^2 + 36s - 24 = 0$; 1 oder $6 \pm \sqrt{12}$.

56. $\frac{1}{2}\left[\sqrt{\frac{2O_1}{\pi} + h^2} - s - \sqrt{s^2 - h^2}\right] = 0{,}55643$.

57. $R = \sqrt{\left[\frac{O_1}{\pi} - r^2 - rs + \frac{s^2}{4}\right]} - \frac{s}{2}$;
$h = \sqrt{s^2 - (R - r)^2} = 7{,}922$.

58. $\frac{1}{2}\left[\sqrt{\frac{2O_1}{\pi} + h^2} - s + \sqrt{s^2 - h^2}\right] = 6$; $(5{,}9999)$.

59. Suche r, dann $h = \sqrt{s^2 - (R - r)^2} = 0{,}156$.

60. $s = \left(\frac{O_1}{\pi} - R^2 - r^2\right) : \left(R + r\right) = 1{,}6$;
$S = s \,.\, R : (R - r) = 8{,}2$.

61. $r^2 = R \,.\, \left(R^2 + RS - \frac{O_1}{\pi}\right) : \left(S - R\right)$; $r = 1{,}4$.

62. $R^3 + R^2 \,.\, S - R \,.\, \left(\frac{O_1}{\pi} - r^2\right) - r^2 S = 0$; $R = 3$.

63. $2r^2\pi + \frac{R^2 - r^2}{R^2} \cdot O = 142.$

64. $S = \left(\frac{O}{\pi} - R^2\right) : R = 80{,}408;$
$r = R \,.\, (S - s) : S = 45{,}201;\ O_1 = 39532.$

65. $R = 1{,}469;\ O_1 = 35{,}5.$

67. $\frac{1}{2} h \sqrt{2}.$ **68.** $r = -\frac{1}{6} b + \sqrt{\frac{a}{3\pi} + \frac{b^2}{36}} = 3{,}455;\ s = 13{,}910.$

69. $p^2 \sqrt{2} : 4\pi.$

70. $r = \frac{s}{4} + \sqrt{\frac{d}{2\pi} + \frac{s^2}{16}} = 2;\ S = s - r = 18.$

71. $s = 2h.$ **72.** $h^2\pi = 207.$ **73.** $2 : 1.$

74. $\frac{1}{2} a\pi \sqrt{2h^2 + a^2} = 100.$

75. $\frac{1}{4} a\pi(\sqrt{2} + 1) \,.\, [a(\sqrt{2} + 1) + \sqrt{4h^2 + a^2(3 + 2\sqrt{2})}].$

76. $\frac{h^2 d\pi}{2(h - d)} = 37{,}654.$

77. $2h : \sqrt{r^2 + h^2};\ r\pi(2h + r + \sqrt{r^2 + h^2}).$

78. $2r\pi(h + r\sqrt{2}) = 500.$

79. Es ist h $= 1\frac{1}{3} r.$

80. Die Höhe des Cylinders ist gleich $\frac{1}{2} h \pm \sqrt{\frac{1}{4} h^2 - \frac{1}{6} s h}.$

81. $n\pi a : \sqrt{81 - 3n^2\pi^2} = 18{,}31.$

82. $(R^2 - r^2)\pi \sqrt{2} = 1244.$

83. $x^2 = \frac{rs\pi - a}{2rs\pi} \cdot h^2;\ x = 13{,}6.$

84. $\frac{\pi}{36} [u^2 + 12h^2 + u\sqrt{u^2 - 12h^2}] = 4011{,}5.$

85. $\sqrt{\frac{M - (b - a)^2}{\pi(\sqrt{a} + \sqrt{b})^2}} = 4.$

86. a) $r\pi\left[r\left(\frac{m}{n} + 1\right) + \left(\sqrt{\frac{m}{n}} + 1\right) W\right] = 818{,}72;$
$W = \sqrt{h^2 + r^2\left(\sqrt{\frac{m}{n}} - 1\right)^2}.$

b) $r\pi\left[r\left(\frac{m}{n} - 1\right) + 2h + \left(\sqrt{\frac{m}{n}} + 1\right) W\right] = 931{,}45;$
$W = \sqrt{h^2 + r^2\left(\sqrt{\frac{m}{n}} - 1\right)^2}.$

87. $M_1 = M_2 = \frac{1}{2}(R + r)s\pi = 1000.$

88. $\frac{1}{2}(P + p)s = 252.$ **89.** $2F\pi.$

90. $s^2\pi : \sqrt{3\sin\alpha^2+1}$. **91.** $a^2\pi\sqrt{2} = 100$.
92. $4a^2\sqrt{2}\pi = 20$. **93.** 30°.

§. 18.

1. $4r^2\pi$. α) 2000. β) 5000. γ) $2500_{,06}$. δ) $3400_{,07}$.
2. $\frac{1}{2}\sqrt{\frac{O}{\pi}}$; α) 2; β) $4{,}5_{0005}$. **3.** $\left(\frac{u^2}{\pi^2}+4p^2\right)\pi = 1412_{,03}$.
4. a) 9281800; b) 9079200. **5.** 56,16 Mark.
6. $\sqrt{r_1^2+r_2^2+r_3^2+r_4^2} = 421$.
7. $16r^2\pi$. **8.** $\frac{4}{15}a^2\pi\sqrt{3} = 2443{,}6$.
9. $a^2\pi = 2$. **10.** $3a^2\pi = 1100$.
11. $\frac{1}{2}A\pi\sqrt{3} = 142$. **12.** 1 : 3 : 9.
13. α) 2 : 3 : 6; β) 1 : 2 : 3. **14.** $\frac{a\sqrt{2}}{\sqrt{\pi}+1} = a(2-\sqrt{2})$.
15. $s = \sqrt{h^2+r^2} = 8{,}81$; $O = \frac{4r^2\pi(s-r)^2}{h^2} = 70{,}086$.
16. $\frac{1}{8}\pi u(u \pm \sqrt{8d^2-u^2}) = 2052{,}7$ oder 1247,3.
17. 3 : 2. **18.** a) $2\sqrt{3} : 3$; b) 1 : 2 : 3.
19. $\frac{2}{3}O\sqrt{3} = 22$.
20. $\sqrt{4+2\sqrt{2}}.(\sqrt{2}+1):[(\sqrt{2}+1)^2+\frac{1}{2}] = 2\sqrt{52+14\sqrt{2}}:17$
21. $\frac{1}{2}\varrho\sqrt{2}$ oder $\frac{1}{3}\varrho\sqrt{3}$.
23. $2h\pi(h \pm p)$; α) 200 oder 75,353; β) 30.
24. $2r\pi(r \mp p)$; α) 400 oder 3523,6; β) 700 oder 6069,6.
25. $r = \sqrt{p^2+\varrho^2}$; $M = 2r\pi(r \mp p)$; α) 49,989 oder 22745; β) 600 oder 6561,9.
26. $h\sqrt{O\pi} = 900$.
27. $h = F : 2r\pi$; $x = rh : (r-h)$; α) 35,12; β) 2,1349.
28. $\frac{2ar^2\pi}{a+r} = 20$. **29.** $3r$. **30.** $\frac{2r(n-1)}{n} = 16$.
31. $b : 2h\pi = 30 : \pi = 9{,}5492$. **32.** $\frac{mh}{2(m-n)}$.
33. $\sqrt{2q(q \mp \sqrt{q^2-p^2})} : p = \sqrt{50} : 7$ oder $\sqrt{50} : 1$.
34. $(m-n) : m$. **35.** $\frac{4}{5}r$. **36.** $\frac{2}{3}r(3-\sqrt{3})$.

37. $x = \frac{ar}{R-r} = 5{,}2$; $O = \frac{2r^2\pi}{a}(a - R + r) = \frac{64}{13}\pi = 15{,}4664$.

38. $2R\pi[\sqrt{R^2 - r_2^2} \pm \sqrt{R^2 - r_1^2}] = 47375$ oder $1633{,}6$.

39. Heisse Zone ungefähr 3670300, jede gemässigte 2427700, jede kalte 378240 Quadratmeilen.

40. Der Abstand des Schnittes vom Mittelpunkt der Halbkugel ist gleich a) $\frac{1}{2}r$, b) $\frac{1}{4}r$.

41. $1 : 4$. **42.** $5 : 13$.

43. $1 : \cos\frac{1}{2}(\alpha_1 - \alpha_2)$ oder $1 : \sin\frac{1}{2}(\alpha_1 + \alpha_2)$.

44. $[2R^2 - p(p + q)] : 2(R^2 - p^2)$.

45. $\frac{\alpha r^2 \pi}{90^0} = \frac{2}{5} r^2 \pi = 2$.

46. a) $0{,}1813 r^2$; b) $0{,}18593 r^2$; c) $0{,}9678 r^2$.

48. $\frac{Oe}{360^0}$. **49.** $\frac{F}{r^2\pi} \cdot 180^0 = 3''$. **50.** $\frac{1}{2}e$.

51. $e_1 : e_2 = r_2^2 : r_1^2$. **53.** $\frac{4F \cdot 180^0}{e} = 21600\,\square^m$.

54. $\sin\frac{1}{2}\alpha = \sqrt{\frac{3h^2 + a^2}{12h^2 + a^2}}$; $F = \frac{r^2\pi}{60^0}(\alpha - 60^0) = 194{,}28$.

55. $3201{,}23$. **57.** $(11 + 3\sqrt{5} - \sqrt{150 + 66\sqrt{5}}) : 4$.

58. $85^0\,53'\,10''$. **59.** $106^0\,15'\,39''$.

60. $4O : (\sin\alpha - 2\cos\alpha + 2)$. **61.** $\frac{1}{4}r\sqrt{22} = 5{,}5$.

62. $4(\sqrt{3}+1) : 8(\sqrt{2}+1) : 2\pi(\sqrt{2}+1) : 8\pi : [3+\sqrt{7+4\sqrt{2}}]\pi : 6\pi$.

63. $(n^2 - 1) : n^2$.

64. $\pi(R + r - s)\sqrt{(R - r + s)(r - R + s)}$;
$(R + r - s) : \sqrt{s^2 - (R - r)^2}$.

65. $\frac{s^2\pi}{h^2}(s^2 + 2h^2 \pm 2sh) = 76030$ oder 22973.

66. $\cos\varphi = \frac{1}{3}$; $\varphi = 70^0\,31'\,43'',3$; $\frac{1}{2}r\sqrt{6}$, wenn r der Radius der kleineren Grundfläche ist.

§. 19.

1. a) 64; b) 729; c) 15625; d) 13,824; e) 0,000027.

2. α) 3,6; β) 1. **3.** 56,7. **4.** $abc : a_1b_1c_1 = 2400$.

5. 403,2 Pfund. **6.** 564. **7.** $1\frac{30}{31} = 1{,}968$.

8. a) 215667 M.; b) 7541,28 M.

9. a) $\frac{O}{6}\sqrt{\frac{O}{6}} = 1{,}728$; b) $\frac{1}{9}\, d^3 \sqrt{3} = 1539{,}6$;
c) $\frac{1}{8}\, u^3 (5\sqrt{2} - 7) = 1000$; d) $F\sqrt{F} : \sqrt[4]{8} = 0{,}216$.

10. $6\sqrt[3]{V^2} = 24\ \square^{m}$. **11.** $\sqrt[3]{V} . \sqrt{3} = 10$. **12.** 8 mal.

13. $\sqrt[3]{a^3 + b^3 + c^3} = 0{,}6$.

14. $\frac{1}{2}\left(b + \sqrt{\frac{4a - b^3}{3b}}\right) = 2{,}3$; $\frac{1}{2}\left(b - \sqrt{\frac{4a - b^3}{3b}}\right) = 1{,}2$.

15. $\sqrt{\frac{O}{6}} \cdot \frac{a}{\sqrt[3]{a^3 + b^3}} = 2{,}5$; $\sqrt{\frac{O}{6}} \cdot \frac{b}{\sqrt[3]{a^3 + b^3}} = 1{,}5$.

16. $\sqrt{\frac{4b - a^3}{12a}} \mp \frac{a}{2} = 7{,}2$ und $7{,}5$. **17.** $\sqrt[3]{bhl}$.

18. $abf + ace + bcd = A = 84$; $aef + bdf + cde = B = 84$;
$g - def = C = 2520$; $\sqrt{4AC + B^2} = D = 924$;
$(D - B) : 2A = E = 5$; $x = aE = 10$; $y = bE = 15$;
$z = cE = 20$.

19. 6 und 2 Decimeter. **20.** $\frac{f^3 ab}{a^2 + b^2} = 23957{,}5$.

21. 16, 13, 10. **22.** $3 . 1 . \frac{3}{4} = 2{,}25$.

23. Sie ist das Doppelte des Inhalts des ersten Prismas.

24. Die Kanten sind bez. gleich c, $\frac{1}{2}(s - c \pm \sqrt{s^2 - 2cs - 3c^2})$.

25. $\frac{1}{2}\, abc = 167{,}5$ Cubm.

26. $\frac{35}{2}(a + b) . cd$ Pf. $= 840$ Pf.

27. $\frac{3}{2}\, a^2 b \sqrt{3} = 1500$. **28.** $2a^2 b (\sqrt{2} + 1) = 448$.

29. $\frac{1}{4}\, a^3 \sqrt{3} = 1000$. **30.** $12\sqrt{3} : 16 : 5\sqrt{10 + 2\sqrt{5}}$.

31. $19\frac{8}{15} = 19{,}533 \ldots \square^{cm}$.

33. $a + b + c + d = 2s = 30$;
$F = \sqrt{(s - a)(s - b)(s - c)(s - d)} = 48$;
$r = \sqrt{(ac + bd)(ad + bc)(ab + cd)} : 4F = \frac{1}{2}\sqrt{130}$;
$h = \frac{mr}{2n} = \frac{13}{4}$; $Fh = 156$.

34. $Fh . \left(1 - \sqrt{\frac{m}{m + n}}\right) = \frac{1}{3}\, hF = 208$.

35. Nach den entsprechenden Formeln für 33 berechne:
$F = 30$; $r = \frac{1}{6}\sqrt{1105}$; $h = \frac{V}{F} = 8$, $R = \sqrt{r^2 + \frac{1}{4}h^2} = 6\frac{5}{6}$.

36. $\frac{1}{2} a^3(27\sqrt{2} - 22\sqrt{3}) = 0{,}039323\, a^3 = 17461.$

37. $a^2 = O : 2\sqrt{3};\ V = 2a^3(5\sqrt{2} - 7) = 23{,}5.$

38. $2ar^2 \sin\alpha \sin\beta \sin\gamma \sin\varphi = 293{,}003.$

39. $\frac{1}{4} a^2 s \sqrt{4(\sin\beta^2 - \cos\alpha^2 + \cos\alpha \,.\, \cos\beta) - 1} = 21350.$

40. $\frac{u^3 \sin 2\alpha}{128 \cos\frac{1}{2}\alpha^6 \cos\beta} \sqrt{\sin(\beta + \alpha)\sin(\beta - \alpha)} = 35393.$

41. $\frac{1}{2} \sin\alpha \sqrt{F^3 \cos\alpha} = 188{,}6;$
$\frac{1}{2}\sqrt{F^3 \cos\alpha}\,(2\cos\alpha - \sin\alpha) = 260{,}92.$

42. $abc\sqrt{1 - \cos\alpha^2 - \cos\beta^2 - \cos\gamma^2 + 2\cos\alpha\cos\beta\cos\gamma}.$

43. $\frac{abc}{\sin\alpha \,.\, \sin\beta \,.\, \sin\gamma}[1 - \cos\alpha^2 - \cos\beta^2 - \cos\gamma^2 - 2\cos\alpha\cos\beta\cos\gamma].$

44. $\frac{2}{5} a^3 \sqrt{2{,}2}.$

45. $\frac{1}{4} d\sqrt{12f^2 - d^4}$ oder $f^2\sqrt{3d^4 - 4f^2} : d^3.$ **46.** $29^0 37' 34''.$

47. $s \sin\varphi \left\{\frac{1}{2} ab \sin\alpha + \right.$
$\left.\frac{1}{4}\sqrt{[(c+d)^2 - (a^2+b^2-2ab\cos\alpha)][(a^2+b^2-2ab\cos\alpha) - (c-d)^2]}\right\}$
$= 1000.$

§. 20.

1. $\frac{1}{3} abh;\ \alpha)\ 1260;\ \beta)\ 0{,}368212;\ \gamma)\ 25{,}5.$

2. $\frac{1}{3} F\sqrt{a^2 - \frac{1}{2}F};\ \alpha)\ 480;\ \beta)\ 40{,}656;\ \gamma)\ 105{,}11;\ \delta)\ \frac{1}{3}F^{\frac{3}{2}}.$

3. Ungefähr 2427780.

4. $\frac{1}{6} ab\sqrt{c^2 - a^2} = 23{,}87.$

5. 6,975 Thlr. = 20,87 M. **6.** $\frac{3}{160} = 0{,}01875.$

7. $\frac{1}{2} a^2 \sqrt{3(b^2 - a^2)};\ \alpha)\ 30492;\ \beta)\ 117612;\ \gamma)\ 1{,}9965.$

8. $\frac{1}{3} aF\sqrt{2} = 1000.$ **9.** $r^3\sqrt{429}.$

10. $\alpha)\ 9520;\ \beta)\ 2016;\ \gamma)\ 0{,}1393.$

11. $3V : h = 52{,}5.$ **12.** $2^m.$ **13.** $\frac{5}{6} a^3.$

14. $\frac{1}{3} F(r \pm \sqrt{r^2 - \frac{1}{2}F}) = 170\frac{2}{3}$ oder $682\frac{2}{3}$ Cubcm.

15. $\frac{16}{3} r^3 \,.\, \frac{m^4 n^2}{(m^2 + n^2)^3} = \frac{9}{16}.$

16. $V_1 = \frac{a^2}{216}\sqrt{3b^2 - a^2} = 23{,}0942;\ V_2 = 17\,V_1 = 392{,}6.$

17. $x : b = \sqrt[3]{6} : 2;\ x = 3,\ b - x = 0{,}30192.$

18. Nach dem goldenen Schnitt.

19. $3\sqrt{5} : (4\sqrt{22} - 3\sqrt{5})$. **22.** $a^3\sqrt{3}$.

23. $O = \frac{1}{4}b(\sqrt{4a^2 - b^2} + \sqrt{4c^2 - b^2}) + a\sqrt{c^2 - a^2} = 10342,4$;
$V = \frac{1}{12}b\sqrt{(4a^2 - b^2)(c^2 - a^2)} = 44431$.

24. $\sqrt[3]{\frac{2}{3}V} = 2$. **25.** $4V : 3h = 30$. **26.** $\frac{1}{2}a$.

27. $27 : 8 : 72$. **28.** $1 : 12$.

29. $\frac{5}{6}a^2b\sqrt{5 + 2\sqrt{5}}$. **30.** $2a$.

31. $h = \frac{1}{2}a = 3^m$; $V = \frac{1}{6}a(a + b)(a + 2b) = 37{,}0872$ Cubm.,
$d = \frac{3ab + 2b^2}{2a} = 9{,}06^{cm}$.

32. $\frac{1}{108}hu^2 \operatorname{cotg} 20^0 = 456{,}01$.

33. $\frac{1}{6}bcs\sqrt{1 - \cos\alpha^2 - \cos\delta^2 - \cos\varepsilon^2 + 2\cos\alpha \cdot \cos\delta \cdot \cos\varepsilon}$.

34. $d = \sqrt{a^2 + b^2 - 2ab \cdot \cos\gamma} = 138$;
$p + q + d = 2s = 384$;
$\sqrt{s(s - p)(s - q)(s - d)} = f = 6624$;
$h = 2F : d = 96$; $V = \frac{1}{3}gh = \frac{2ab\sin\gamma \cdot F}{3d} = 139104$.

35. $\frac{1}{24}na^3 \cdot \operatorname{cotg}\left(\frac{180^0}{n}\right)^2 \cdot \operatorname{tang}\alpha = 905$.

36. $\frac{1}{12}c^2l \cdot \sin 2\alpha \cdot \sin\varphi = 920$.

37. $a = \sqrt[3]{3\sqrt{2}V \operatorname{cotg}\alpha} = 4{,}641$;
$s = \frac{a\sqrt{2}}{2\cos\alpha} = \sqrt[3]{\frac{3V}{\sin 2\alpha \cdot \cos\alpha}} = 8{,}820$.

38. $a \pm b = \sqrt{4s^2\cos\alpha^2 \pm \frac{6V}{s \cdot \sin\alpha}}$; $a = 3{,}0002$,
$b = 2$; $(1{,}9999)$.

39. $\operatorname{tang}\varphi = 24V \cdot \sin\frac{180^0}{n} : na^3 \cdot \operatorname{cotg}\frac{180^0}{n}$.

40. $\frac{9}{16}R^3\sqrt{3} = 36\sqrt{3}$; $\operatorname{tang}\varphi = 2$; $\varphi = 63^0\,26',1$.

41. $V = \frac{1}{3}a^3$; $O = a^2(\sqrt{5} + 1)$; Winkel an den Grundkanten: $\operatorname{tang}\alpha = 2$; $\alpha = 63^0\,26'\,5'',7$; Winkel an den Seitenkanten: $\sin\frac{1}{2}\beta = \frac{1}{10}\sqrt{15}$; $\beta = 45^0\,34'\,22''$.

42. $V = \frac{1}{6}a^3$; $O = a^2(\sqrt{2} + 1)$; $\alpha = 45^0$; $\sin\frac{1}{2}\beta = \frac{1}{4}\sqrt{3}$;
$\beta = 51^0\,19'\,4''$.

43. $V = \frac{1}{3}a^3$; $O = a^2(2 + \sqrt{2})$; $\alpha = 90^0$ und 45^0;
$\beta_1 = \beta_2 = \beta_3 = 90^0$; $\beta_4 = 120^0$.

44. $a = 2\sqrt[3]{k \cdot \operatorname{cotg} \alpha} = 2{,}82847 = \sqrt{8}$;

$F = \frac{a^2\sqrt{3}}{4\cos\alpha} = 6$; $O = \frac{a^2\sqrt{3}}{4\cos\alpha}(1+\cos\alpha+2\sin\alpha) = 9{,}26188$;

$\varrho = \frac{a\sqrt{3}}{4\cos\alpha}\operatorname{tang}\frac{1}{2}\alpha(1+\cos\alpha-\sin\alpha) = 0{,}8355$.

45. $k = u : (p+q+r)$;

$a = pk = 20,\ b = qk = 48,\ c = rk = 52$;

$s = \frac{1}{2}u = 60$;

$F = \sqrt{s(s-a)(s-b)(s-c)} = 480$;

$\varrho = F : s = 8$; $p = \sqrt{h^2+r^2} = 10$;

$V = \frac{1}{3}Fh = 960$; $O = F + ps = 1280$;

$\operatorname{tang}\alpha = h : \varrho$; $\alpha = 36^0\,52',2$.

§. 21.

1. α) 1887; β) 135,9; γ) 0,0441.

2. $\frac{1}{3}p\left(G\sqrt{\frac{G}{g}} - g\right)$; α) 326403; β) 0,04965; γ) 3899,3136.

3. $a+b+c = 2s = 16$; $G = 12$; $g = \frac{a_1^2}{a^2}G = 9{,}72$;

$R = \frac{abc}{4G} = \frac{25}{8}$, $r = \frac{a'}{a}R = \frac{45}{16}$;

$h = \sqrt{d^2-(R-r)^2} = \frac{3}{4}$; $V = 8{,}13$.

4. $V = \frac{1}{12}\sqrt{3} \cdot h \cdot (a^2+ab+b^2) = 543$;

$p = \frac{ah}{b-a} = 1{,}319657$.

5. $\frac{3V}{h} - \frac{1}{2}G - \sqrt{3G\left(\frac{V}{h} - \frac{1}{4}G\right)}$;

α) $129 - 72\sqrt{3} = 4{,}294$; β) 26,01.

6. $\frac{Vg\sqrt{g}}{G\sqrt{G} - g\sqrt{g}} = 3{,}743513$.

7. $u^2 : (U^2+Uu+u^2) = M = \frac{1}{21}$; $a+b = \frac{1}{2}u$;

$a - b = \sqrt{\frac{1}{4}u^2 - \frac{12VM}{h}} = 1$; $A = \frac{aU}{u}$, $B = \frac{bU}{u}$,

$a = 3,\ b = 2,\ A = 12,\ B = 8$; $c = \frac{1}{2}\sqrt{117}$.

8. $g = \frac{3V}{h} - \frac{1}{2}G - \sqrt{3G\left(\frac{V}{h} - \frac{G}{4}\right)} = 0{,}05$;

$p = \frac{h\sqrt{g}}{\sqrt{G} - \sqrt{g}} = 0{,}48$; $x = \frac{1}{3}gp = 0{,}008$.

9. $V \cdot \frac{G^3 + Gg\sqrt{Gg}}{G^3 - g^3}$.

10. $G = \frac{1}{2}\left[S + \sqrt{\left(\frac{6V}{h} - S\right)\left(3S - \frac{6V}{h}\right)}\right] = 2{,}89;$

$g = \frac{1}{2}\left[S - \sqrt{\left(\frac{6V}{h} - S\right)\left(3S - \frac{6V}{h}\right)}\right] = 1{,}69;$

$p = \frac{h\sqrt{g}}{\sqrt{G} - \sqrt{g}} = 4{,}875;\ V_1 = \frac{1}{3}\frac{hG\sqrt{G}}{\sqrt{G} - \sqrt{g}} = 6{,}14125.$

11. $\frac{1}{2}\left(\frac{4V}{h} + d\right) \pm \sqrt{\frac{V^2}{h^2} - \frac{1}{12}d^2}$ und

$\frac{1}{2}\left(\frac{4V}{h} - d\right) \pm \sqrt{\frac{V^2}{h^2} - \frac{1}{12}d^2}$.

12. $\sqrt{\frac{3p}{1000hs} - \frac{3}{4}a^2} - \frac{1}{2}a = 1{,}05557.$

13. 9^{dc}, 16^{dc}, $303{,}1733\,\square^{dc}$.

14. $\frac{5}{24}\sqrt{10 + 2\sqrt{5}} \cdot \sqrt{s^2 - (r - \varrho)^2} \cdot (r^2 + r\varrho + \varrho^2) = 66{,}57401.$

15. $m^3 : (n^3 - m^3)$. **16.** $1 : (\sqrt[3]{n} - 1) = 1 : 1.$

17. a) $\sqrt[3]{2} : 1$, b) $1 : (\sqrt[3]{2} - 1)$.

18. $\frac{1}{\sqrt[3]{2}} \cdot \sqrt{a^2 - \frac{1}{4}(b^2 + c^2)} = 2{,}00005.$

19. $\frac{h}{\sqrt[3]{3}} = 4{,}8536;\ h \cdot \frac{\sqrt[3]{2} - 1}{\sqrt[3]{3}} = 1{,}2615;\ h \cdot \frac{\sqrt[3]{3} - \sqrt[3]{2}}{\sqrt[3]{3}} = 0{,}8849.$

20. $\frac{7a^2 + 4ab + b^2}{a^2 + 4ab + 7b^2} = \frac{217}{271}$.

21. $x = \sqrt[3]{\frac{1}{2}(a^3 + b^3)};\ m : n = (x - b) : (a - x)$.

22. $H - \frac{1}{2}h \pm \sqrt{\frac{H^2V}{hG} - \frac{1}{12}h^2}$; 34.

23. $\frac{4}{147}r^2h(10 - \sqrt{2})$.

24. $\frac{19}{162}a^2\sqrt{b^2 - \frac{1}{2}a^2} = 67\frac{5}{9}$. **25.** $\frac{2}{3}h(R^2 + Rr + r^2)$.

26. $r = \frac{1}{2}\sqrt{a^2 + b^2} = 1{,}95;\ p = \frac{2r^2h}{r^2 + h^2};\ \log p = 0{,}26662;$

$V = \frac{1}{3}pab\left[1 - \frac{r^2 - h^2}{r^2 + h^2} + \left(\frac{r^2 - h^2}{r^2 + h^2}\right)^2\right]$

$= \frac{1}{3}\frac{pab}{h^2}(3h^2 - 3hp + p^2) = 2{,}6023.$

27. $\frac{1}{24} a^3 \sqrt{38430 - 3510\sqrt{5}}$.

28. $\frac{1}{12} a^3 \sqrt{470 + 210\sqrt{5}}$.

29. $2 : 1$ oder $\sqrt{13} : 5$. **30.** Sie sind gleichgross.

31. $G = 21$; $g = a_1^2 . G : a^2 = \frac{21}{25}$; $h = s . \sin\varphi = 25$; $V = 217$.

32. $\operatorname{tang}\alpha = \frac{3V\sqrt{2}}{a^3 - b^3} = \frac{9}{4}\sqrt{2}$; $\operatorname{tang}\beta = \frac{6V}{a^3 - b^3} = \frac{9}{2}$, $\alpha = 72^0\,33'\,13''$, $\beta = 77^0\,28'\,16''$.

33. $G = \frac{3}{2}\sqrt{35}$, $g = \frac{49}{121} G$, $\alpha = 33^0\,40'$.

34. $G = 4{,}5\, a^2 \operatorname{cotg} 10^0 = 12{,}81$; $\varrho_1 = \frac{1}{2} a : \sin 10^0 = 2{,}04$; $p = \sqrt{r^2 - \varrho_1^2} = 2{,}53$; $q = h - p = 3{,}23$; $\varrho_2 = \sqrt{r^2 - q^2} = 0{,}36$; $g = G\varrho_2^2 : \varrho_1^2 = \frac{6{,}57}{17}$; $V = 29{,}542$.

35. $\frac{3}{2} r^3 . \frac{3n^2 - 3n + 1}{n^3} . \operatorname{tang}\varphi . \sin 40^0 = 1900$.

§. 22.

1. Tetraëder: $\frac{1}{12} a^3 \sqrt{2}$, Oktaëder: $\frac{1}{3} a^3 \sqrt{2}$, Ikosaëder: $\frac{5}{12} a^3(3+\sqrt{5})$, Hexaëder: a^3, Dodekaëder: $\frac{1}{4} a^3(15+7\sqrt{5})$.

2. $1 : \sqrt{2}$. **3.** $a^2 \sqrt[3]{3} = 100$. **4.** $\frac{1}{8} h^3 \sqrt[3]{3} = 125$.

5. $\frac{u^3\sqrt{6}}{32\pi^3} = 835$. **6.** $\frac{1}{3} a^3$. **7.** $\sqrt{3} . \sqrt[3]{36 V^2} = 70{,}7$.

8. $\frac{1}{6} a^3$. **9.** $\sqrt{2} : \sqrt[4]{3}$. **10.** $4r^3\sqrt{3}$.

11. $\frac{5}{3} a^3 \sqrt{2}$. **12.** $9 : 64$. **13.** $3\sqrt[3]{4} - 2 = 2{,}7622$.

14. $a\sqrt[3]{6\sqrt{2} + 5} = 169$.

15. $- h(\sqrt[3]{2} - 1)$, also auf der Höhe selbst, statt auf der Verlängerung.

16. $1 : 27$.

17. a) $\sqrt{3} : 9$; b) $\sqrt{3(125 + 58\sqrt{5})} : 45$; c) $\sqrt{3} : 9$; d) $\sqrt{3(85 - 38\sqrt{5})} : 45$.

18. $\frac{2}{3} V\sqrt{3} = 384{,}9$. **19.** $8r^3\sqrt{3}$; $4r^3\sqrt{3}$.

20. $\frac{1}{3} r\sqrt[3]{2} = 2{,}65$. **21.** $\frac{2}{81} r^3 \sqrt{2}$.

22. $121{,}5 \,.\, a^3 = 36$. **23.** $2r^3\sqrt{6}$.

24. $27(5\sqrt{2} - 7) : 1$.

25. Es ist ein regelmässiges Tetraëder, und die Volumina verhalten sich wie 1 : 4.

26. $26 : 7$. **27.** $1 : (\sqrt[3]{4} - \frac{3}{4}) : (\sqrt[3]{9} - \frac{3}{4}\sqrt[3]{4} + \frac{3}{4})$.

28. $\frac{1}{27} a^3\sqrt{2}$ und $\frac{5}{108} a^3\sqrt{2}$. **29.** $r \,.\, \sqrt[6]{2}$.

30. $(5 - \sqrt{5}) : 4$. **31.** $48(\sqrt{5} - 2) : 5$.

32. $\frac{5}{12} a^3 \sqrt[4]{108 \,.\, (49 + 38\sqrt{5})}$.

33. $3(25 + 9\sqrt{5}) : 49$. **34.** $3(5 - \sqrt{5}) : 10$.

35. $(7\sqrt{5} + 15) : 6 = 5{,}108744 : 1$ oder $\frac{3\sqrt{5} - 5}{6}\left(\frac{\sin 54^0}{\sin 18^0}\right)^3$.

36. $\frac{2}{3}\sqrt{\frac{F^3}{3\sqrt{6}}(\sin\alpha + \sqrt{2} \,.\, \cos\alpha)^3} = 64$.

37. $\frac{1}{3} a^3 (\sqrt{2} + \operatorname{tang}\alpha) = 729$.

38. $\operatorname{tang}\alpha = \sqrt{2}$, $\alpha = 54^0\,44'\,7'',8$.

§. 23.

1. a) 765; b) 999; c) 4,24. **2.** $9\pi = 28{,}274334$.

3. 64,34. **4.** $u^2 h : 4\pi = 56{,}5$. **5.** $b : a$.

6. $h = V : r^2\pi$; α) 3; β) 4; γ) $150^{mm}{,}43$.

7. $2\sqrt{\frac{V\pi}{h}} = 400$.

8. $r = \sqrt[3]{\frac{nV}{2m\pi}} = 2{,}331$; $h = \frac{2m}{n} r = \sqrt[3]{\frac{4m^2V}{n^2\pi}} = 2{,}797$.

9. $AB : \pi\delta\varepsilon : \left(2\sqrt{\frac{A}{\pi\delta}} - \varepsilon\right) = 192$.

10. $r(2h - r) : h = 4{,}8$. **11.** $r : h = 3 : 2$, $x = r : \sqrt{3}$.

12. $r : h = 1 : 2$, $x = r\sqrt{3}$. **13.** $(R + r)(R - r)\pi h = 800$.

14. $R - \sqrt{R^2 - \frac{V}{\pi h}} = 2$.

15. $r = \frac{2V}{M} = 2$, $h = \frac{M^2}{4V\pi} = 3$.

16. $\frac{M}{4}\sqrt{\frac{nM}{m\pi}} = 100$.

17. $(2m+n)\sqrt[3]{\frac{2V^2\pi}{m^2n}} = 64\pi = 201{,}0619.$

18. $\frac{O-M}{2} \cdot \frac{M}{\sqrt{2(O-M)\pi}} = 1000.$ **19.** $4^m.$

20. $V + 2\sqrt{VV_1} + V_1 = 2{,}43.$ **21.** $F = \frac{d^2}{2\pi},\ V = \frac{d^3\sqrt{2}}{16\pi}.$

22. $r\sqrt{\frac{26}{1109}} = 5{,}63.$ **23.** 232,973 Kgr.

24. $\frac{1}{2}d - \sqrt{\frac{1}{4}d^2 - \frac{p}{1000\pi\,.\,as}} = 0^m{,}00355;$

$250\,ad^2\pi - \frac{p}{s} = 520{,}14$ Kgr.

25. $(a^2 - \frac{1}{4}d^2\pi)h = 400.$

26. $12\sqrt{101} : 5\sqrt{577} : 13;$

$(48\sqrt{101} + 101) : (20\sqrt{577} + 577) : 728;$

$1212 : 2885 : 338.$

27. $\frac{a^3\sqrt{2}}{12r^2\pi} = 0{,}29307.$ **28.** $\frac{3}{4}r^3\pi.$

29. $2r^3\pi\sqrt{2\,.\,\frac{m+n\pm\sqrt{m^2+2mn-4n^2}}{5m}\,.}$

$\overline{\frac{(4n-m)\mp\sqrt{m^2+2mn-4n^2}}{5m}} = 783{,}12.$

30. $1:6.$ **31.** $16:27.$ **32.** $4:3.$

33. (Vergl. §. 19, 33.) $F = 216;\ r^2 = \frac{468\,.\,582\,.\,432}{16F^2},\ V = 4304\frac{3}{8}.$

34. $G = \sqrt{s(s-a)(s-b)(s-c)} = 330;$

$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} = 6{,}6;\ h = K : G;\ V = 100.$

35. $r^2h\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = 36{,}609;\ r^2h\left(\frac{5\pi}{6} + \frac{\sqrt{3}}{4}\right) = 1233{,}03.$

36. $\frac{1}{8}F\sqrt{\frac{1}{2}F\pi} = 1{,}5.$

37. $a^2\pi;\ a^2(2\pi - \sqrt{3});\ \frac{1}{2}a^3(\pi - \sqrt{3}).$

38. $\frac{1}{2}(S+s)r^2\pi = 295.$ **39.** $6ar^2\pi.$

40. $ar^2\pi\,.\,\sin\alpha = 214.$ **41.** $a\sin\alpha\sqrt[3]{4:\pi} = 1{,}35.$

42. $a\sqrt{\frac{h\sin\alpha}{s\pi\sin\beta}}.$ **43.** $f^2\pi : 4a\sin\alpha = 105\ (104{,}998).$

44. $\cos\varphi = d : r = \cos 40^0$;
$rh\left(\frac{r\pi\varphi}{180^0} - d \sin\varphi\right) = 48000$ (47998).

45. $\frac{V\pi}{2 \sin\alpha \,.\, \sin\beta \,.\, \sin\gamma} = 2{,}21$.

46. Cylinder: $2r^3\pi \cdot \frac{\sin\beta \,.\, \sin\gamma}{\sin\alpha} = 75760$;
Prisma: $4r^3 \sin\beta^2 \sin\gamma^2 = 27640$.

47. $\frac{V}{2\pi} \,.\, n \,.\, \sin\frac{360^0}{n} = 79$. 48. $\operatorname{tang}\alpha = \frac{q}{p+q} \cdot \frac{h}{r}$; $\alpha = 45^0$.

49. $\frac{a\,d\,\alpha\,\pi}{360^0}\left(d + \frac{c^2 + \frac{1}{4}b^2}{c}\right) = 71$; $\sin\alpha = \frac{4bc}{b^2 + 4c^2}$.

§. 24.

1. α) 235,62; β) 1,5928; γ) 2537940.
2. $\sqrt{3V : \pi h} = 3$. 3. $3V : r^2\pi = 22{,}6$.
4. 1 Thlr. 17 Sgr. 7 Pf.
5. $\frac{1}{3} r^2\pi\sqrt{s^2 - r^2}$. α) 15394; β) 28,743; γ) 32.
6. $\frac{1}{3}\pi h(s^2 - h^2) = 700$. 7. $\sqrt{r^2 + \frac{9V^2}{r^4\pi^2}} = 7{,}45$.
8. $\frac{1}{3} r\sqrt{M^2 - r^4\pi^2}$; α) 247,537; β) 0,24618.
9. $\frac{M^2}{3s^2\pi}\sqrt{s^2 - \frac{M^2}{s^2\pi^2}} = 20$.
10. $\frac{1}{3}\pi h\left(\sqrt{\frac{M^2}{\pi^2} + \frac{h^4}{4}} - \frac{h^2}{2}\right) = 91{,}292$.
11. $\sqrt{r^6\pi^2 + 9V^2} : r = 185$.
12. $\sqrt{3V(3V + \pi h^3)} : h = 89{,}46$.
13. $\frac{1}{3} r\sqrt{O^2 - 2Or^2\pi} = 2$.
14. $\frac{\pi}{3}\left[\frac{O}{\pi} + \frac{s^2}{2} - s\sqrt{\frac{O}{\pi} + \frac{s^2}{4}}\right] \cdot$
$\sqrt{\frac{s^2}{2} - \frac{O}{\pi} + s\sqrt{\frac{O}{\pi} + \frac{s^2}{4}}} = 9509$.
15. $\frac{1}{3}\frac{O^2 h}{2O + h^2\pi} = 111$.
16. $\sqrt{\frac{3V\pi}{h}}\left(\sqrt{\frac{3V}{\pi h} + h^2} + \sqrt{\frac{3V}{\pi h}}\right) = 2099{,}43$.
17. $\frac{1}{3} r^3\pi\sqrt{3} = 3500$. 18. $\frac{1}{3} F\sqrt{f\pi} = 50$.
19. $\frac{1}{3} a\pi\sqrt{\frac{1}{3} a\sqrt{3}} = 26$.

20. $O = \frac{9}{4} r^2 \pi$; $V = \frac{3}{8} r^3 \pi$. **21.** $0{,}9\,h$.

22. $r = \frac{1}{2}(\sqrt{2s^2 - d^2} - d) = 39$.
$M = \frac{1}{2} s\pi (\sqrt{2s^2 - d^2} - d) = 10904{,}4$;
$V = \frac{1}{12}\pi (s^2 - d^2)(\sqrt{2s^2 - d^2} - d) = 127421$.

23. $r = \sqrt{\frac{1}{2}\sigma^2 \pm \sqrt{\frac{1}{4}\sigma^4 - \frac{6K\sigma}{\pi}}} = 11{,}866$ oder $4{,}8$;
$h = \frac{1}{2\sigma}(\sigma^2 - r^2) = 0{,}9$ oder $5{,}5$;
$s = \frac{1}{2\sigma}(\sigma^2 + r^2) = 11{,}9$ oder $7{,}3$;
$M = \frac{r\pi}{2\sigma}(\sigma^2 + r^2) = 443{,}61$ oder $110{,}08$;
$O = \frac{r\pi}{2\sigma}(\sigma + r)^2 = 885{,}94$ oder $182{,}46$.

24. $\frac{m h^2 \pi}{n^2}\sqrt{m^2 - n^2}$, $\frac{h^3\pi}{3n^2}(m^2 - n^2)$.

25. Beide wie $a : b$. **26.** $2\sqrt{3} : 6 : 3$.

27. $r^6 - \frac{M^2}{\pi^2} r^2 + \frac{9V^2}{\pi^2} = 0$; $r = 1{,}5$, $h = 2$.

28. $\frac{M}{3n}\sqrt{\frac{n^2 - 1}{n\pi}} \,.\, M$.

29. $r = \sqrt[3]{\frac{3V}{\pi\sqrt{n^2 - 1}}} = 1$, $h = r\sqrt{n^2 - 1} = \sqrt{3} = 1{,}73205$;
$s = nr = 2$.

30. $\frac{1}{3} m^2 h^3 \pi : (n^2 - m^2) = 1231{,}51$.

31. $\frac{97 \pm 20\sqrt{22}}{3}$ mal. **32.** $9(R^2 + H^2) = R^2 . H^2$.

33. $\frac{1}{192} a^3 \pi \sqrt{15} = 2250$. **34.** $\frac{1}{3}\sqrt{35 F^3 : \pi} = 100$.

35. $2\sqrt[3]{V : \pi\sqrt{11}} = 48$.

36. $\sqrt[3]{3V : (3V + h^3\pi)} \,.\, 360^0 = 45^0$.

37. $138^0 27' 41'',54$. **38.** $K - \frac{p\pi}{3q}\left(\sqrt[3]{\frac{3qK}{p\pi}} - b\right)^3 = 124$.

39. $R = \frac{1}{2}\left[\sqrt{\frac{1}{3}\left(\frac{4V\sqrt{3}}{\pi d} - d^2\right)} + d\right] = 4$;
$r = \frac{1}{2}\left[\sqrt{\frac{1}{3}\left(\frac{4V\sqrt{3}}{\pi d} - d^2\right)} - d\right] = 3$.

40. $\sqrt{6} : 2$. **41.** $\sqrt[3]{18} : 3$.

42. $\frac{1}{2} r \sqrt{2}$; der Mantel wird ebenfalls halbirt.

43. $r \sqrt[3]{\frac{1}{2}}$. **44.** $x^3 + hx^2 = h^3$.

45. $O = \pi \sqrt{s+r} \,.\, (r\sqrt{s+r} + \varrho \sqrt{s-r}) = 87{,}9646$;
$V = \pi \sqrt{s^2 - r^2} \,.\, (\frac{1}{3} r^2 - \frac{1}{2} \varrho^2) = 31{,}4159$.

46. $\frac{3}{4} h$. **47.** $\frac{Rr\pi}{(R+r)^2}(r\sqrt{R^2+h^2} + R\sqrt{r^2+h^2})$; $\frac{R^2 r^2 h\pi}{3(R+r)^2}$.

48. $O = 9\sqrt{3} \,.\, \sqrt[3]{3a^2 : \pi}$; $V = 3a\sqrt{3} : \pi$.

49. $\frac{1}{6} K\pi \sqrt{3} = 552$.

50. Nach den Formeln für §. 19, 33 ist $F = 27$,
$r = \sqrt{41 : 481} : 32$, $V = 192{,}59$.

51. $\frac{1}{27} a^3 \pi \sqrt{6}$; $\frac{1}{108} a^3 \pi \sqrt{6}$; $4 : 1$. **52.** $\sqrt[3]{3 \,.\, \pi} : 2$.

53. Die Volumina wie $1 : 7 : 12$, die Oberflächen wie $1 : 3$. Die des Doppelkegels für $r : h = \sqrt{15} : 1$, die des anderen Körpers für $r : h = \sqrt{7} : 3$. **54.** $8 : 1$.

55. $O = \frac{1}{4} r^2 \pi (3 + 2\sqrt{3}) = 3750080$ □ Meilen,
$V = \frac{1}{8} r^3 \pi = 249300000$ Cub.-Ml.

56. $\frac{2\pi}{3} \cdot \frac{Rr^4}{r^2 - R^2} = 96$.

57. $r = \sqrt[3]{\frac{3n^3 K}{\pi(n+m)(n^2-m^2)}} = 4{,}0966$;
$\varrho = \frac{r}{n} \sqrt{n^2 - m^2} = 3{,}0535$.

58. $\frac{R^2 c\pi (c + R - r)}{(R-r)(c-R+r)}$; $\frac{\pi}{3} \cdot \frac{R^3 (c+R-r)^2}{(R-r)(c-R+r)}$.

59. $(2\pi - 3\sqrt{3}) : (10\pi + 3\sqrt{3}) = 108703 : 3661208$.

60. $\frac{1}{3} r^3 \pi \operatorname{cotg} \frac{1}{2} \alpha = 165$. **61.** $\frac{1}{3} s^3 \pi \,.\, \cos\beta^2 \,.\, \sin\beta = 638$.

62. $S + s + 2r = \sigma$; $F = \sqrt{\sigma(\sigma - S)(\sigma - s)(\sigma - 2r)}$;
$h = F : r$; $V = \frac{1}{3} r^2 \pi h$, oder
$\cos\beta = (S^2 + 4r^2 - s^2) : 4rS$; $h = S \,.\, \sin\beta$, etc.
α) $F = 1020$; $h = 40$; $\cos\beta = \frac{21}{29}$; $\sin\beta = \frac{20}{29}$;
$V = 27237{,}5$. β) $F = \frac{9}{50}$; $h = \frac{36}{25}$; $\cos\beta = \frac{7}{25}$;
$\sin\beta = \frac{24}{25}$; $V = 0{,}023562$.

63. $r^2 = \frac{1}{2}(S^2 + s^2 - a^2) = 64$; h (wie in 62) $= \frac{3}{4}\sqrt{231}$;
$V = \frac{1}{3} r^2 \pi h = 764$.

64. $\sqrt[3]{\frac{9V^2\pi}{\cos\alpha \,.\, \sin\alpha^2}} = 140$.

65. $243{,}89\,\square^{dm}$. **66.** $\frac{\pi}{3}\cdot\frac{a^3 \sin\frac{1}{2}\alpha^2 \,.\, \sin\frac{1}{2}\beta^2}{\sin\frac{1}{2}(\beta-\alpha)^2} = 36.$

67. $\frac{p\pi}{12\cos\alpha \,.\, \sin\alpha^2}\cdot(3d^2\sin\alpha^2 - 6dp\sin\alpha + 4p^2) = 264.$

68. $h = \frac{p}{2\pi}\operatorname{cotg}\frac{1}{2}\alpha = 1{,}1789;\ s = \frac{p}{2\pi\sin\frac{1}{2}\alpha} = 9{,}0768;$

$M = \frac{p^2}{4\pi\sin\frac{1}{2}\alpha} = 256{,}635;$

$V = \frac{p^3}{24\pi^2}\operatorname{cotg}\frac{1}{2}\alpha = 100\ (99{,}998).$

69. $\operatorname{tang}\varphi = \frac{2\sin\alpha \,.\, \sin\beta}{\sin(\beta-\alpha)};\ \varphi = 59^0\ 52'\ 55'',8;$

$V = \frac{\pi}{3}\cdot a^3\cdot\frac{\sin\alpha^2 \,.\, \sin\varphi}{\sin(\varphi-\alpha)^2} = 66{,}9.$

70. $\operatorname{tang}\varphi = \frac{2\sin\alpha \,.\, \sin\beta}{\sin(\beta-\alpha)};\ \varphi = 49^0\ 23'\ 46'',0;$

$V = \frac{\pi}{3}\cdot S^3\cdot\frac{\sin\alpha^2 \,.\, \sin(\varphi-\alpha)}{\sin\varphi^2} = 59{,}89.$

71. $\sin\beta = \frac{S\sin\varphi\sin\alpha}{a},\ V = \frac{a^2 S\pi\sin(\varphi+\beta)}{12\sin\alpha^2} = 76{,}852.$

72. $D = \frac{1}{4}V\pi = 94{,}91;\ b = \sqrt[3]{\frac{3V}{2\cos\alpha^2\sin\alpha}} = 9{,}1538.$

73. $V\pi \,.\, \operatorname{tang}\frac{1}{2}\alpha^2 \,.\, \operatorname{cotg}\alpha = 400.$

74. $r^2 = \frac{(s-a)(s-b)(s-c)}{s} = \frac{225}{4};$

$\frac{1}{3}r^2 d\pi\sin\delta = 8000{,}2.$

75. $\frac{\pi}{3}\cdot\frac{h^3\operatorname{tang}\frac{1}{2}\beta^2}{\sin\alpha^2} = 12.$

76. $\sin\varphi = \frac{t}{4r}\cdot\sin\alpha \pm \sin\frac{1}{2}\alpha\sqrt{1+\frac{t^2}{4r^2}\cos\frac{1}{2}\alpha^2};$

$V = \frac{1}{3}\pi r^2 t\sin\varphi.$

77. $2R\pi : \sqrt{3} \,.\, \sin\alpha^3 = 32{,}31.$

78. $2{,}6399 : 1{,}2255 : 5{,}9683 : 3{,}5810.$

§. 25.

1. $\alpha)$ 500; $\beta)$ 0,02.

2. $\sqrt{\frac{3V}{\pi h} - \frac{3}{4}R^2} - \frac{R}{2} = 2.$

3. $3V : \pi(R^2 + Rr + r^2) = 100.$

4. $\frac{\pi}{3}(R^2 + Rr + r^2) \,.\, \sqrt{s^2 - (R-r)^2} = 263365.$

5. $\frac{1}{12}\pi h \cdot \left(\frac{3M^2}{s^2\pi^2} - s^2 + h^2\right) = 128{,}491.$

6. $\frac{1}{3}\frac{M^2}{s^2\pi} \cdot \frac{p^2+pq+q^2}{(p+q)^2} \cdot \sqrt{s^2 - \frac{M^2}{s^2\pi^2} \cdot \frac{(p-q)^2}{(p+q)^2}} = 334.$

7. $\sqrt{\frac{3V}{\pi h(m^2+mn+n^2)}} = A;\ r = n\,.\,A;\ R = m\,.\,A;$
$\alpha)\ r = 6,\ R = 18;\ \beta)\ r = 4,\ R = 6.$

8. $R = \sqrt{\frac{V}{\pi h} - \frac{1}{12}d^2} + \frac{1}{2}d = 7;$
$r = \sqrt{\frac{V}{\pi h} - \frac{1}{12}d^2} - \frac{1}{2}d = 5.$

9. $h = \frac{a^2-b^2}{2b} = 264;\ s = \frac{a^2+b^2}{2b} = 265;$
$r = \sqrt{\frac{V}{\pi h} - \frac{1}{12}a^2} - \frac{a}{2} = 107;$
$R = \sqrt{\frac{V}{\pi h} - \frac{1}{12}a^2} + \frac{a}{2} = 130;$
$M = 2s\pi\sqrt{\frac{V}{\pi h} - \frac{1}{12}a^2} = 197309.$

10. $S + s + 2(R-r) = 2\sigma;$
$F = \sqrt{\sigma(\sigma - S)(\sigma - s)(\sigma - 2R + 2r)};$
$h = F : (R-r)$ u. s. w. $\alpha)$ 95818; $\beta)$ 3808450;
$\gamma)$ 6,21614.

11. $F = 0{,}36;\ h = 0{,}8;\ R = 1{,}3;\ r = 0{,}85.$

12. $\frac{2h-3e}{3e-h} = m;\ r = \sqrt{\frac{3V}{\pi h} : (m^2+m+1)} = 4;\ R = mr = 5.$

13. $\frac{g}{3} \cdot \sqrt{\frac{3g}{\pi}} \cdot (n + \sqrt{n} + 1) = 700.$

14. $M = \frac{1}{3}d^2\pi;\ 1:1,\ V = \frac{1}{72}d^3\pi(3\sqrt{3} - 1).$

15. Der Sector eines Kreisrings mit den Radien $\sigma = \frac{rs}{R-r} = 24$,
$S = \frac{Rs}{R-r} = 30$ und dem Centriwinkel
$\alpha = \frac{R-r}{s} \cdot 360^0 = 300^0$, wenn $s = \sqrt{h^2 + (R-r)^2}$,
$h = 3V : \pi(R^2 + Rr + r^2)$ ist.

16. $\varrho = \sqrt{\frac{1}{2}(R^3+r^3)} = 8{,}25483;\ x = \frac{\varrho - r}{R-r} \cdot h = 7{,}81159.$

17. $12a^2hq : [p\pi\sqrt{s^2 - \frac{1}{4}(D-d)^2} \cdot (D^2 + Dd + d^2)] = 100.$

18. $\frac{a}{2\pi} \cdot \left(\frac{U^2 + Uu + u^2}{6} - \frac{u^2}{\pi}\right) = 0{,}0678033$ Cbm.

19. $\frac{1}{3} h (R^2 + Rr + r^2)(\pi - 2) = 31{,}2343.$

20. $\frac{7\pi a^3 \sqrt{6}}{864}$. **21.** $\frac{V\sqrt{3}}{2\pi} = 20000.$ **22.** $\frac{1}{4} V\pi = 71.$

23. $\frac{1}{12}\pi h(R - r)^2.$

24. $\frac{1}{3}\pi(R^2 + Rr + r^2 - 3x^2)\sqrt{s^2 - (R - r)^2} = 144.$

25. $4 : 1.$ **26.** $\frac{3(D^2 - d^2)l}{a^2 + ab + b^2} = 31\frac{53}{63}.$

27. $r = \sqrt{\frac{V}{\pi h} - \frac{1}{12} d^2} - \frac{1}{2} d = 4;\ \frac{rh}{d} = 8.$

28. $\frac{r^2\pi h}{6}(3 - \sqrt{2}).$ **29.** $\frac{1}{2} r(\sqrt{3} - 1) = 1.$

30. $\frac{1}{2} r(\sqrt{3} - 1) = 5;$
$\frac{r(\sqrt{3}+1)}{2} \cdot \pi \cdot \sqrt{4h^2 + \frac{3}{2}r^2(2 - \sqrt{3})} = 775{,}517.$

31. $x = \frac{a(\sqrt{3h(4p - h)} - h)}{4(h - 3p)};$
$n\pi \left\{(\frac{1}{2} a + x)\sqrt{h^2 + (\frac{1}{2} a - x)^2} + 2px\right\}.$

32. $\frac{1}{24} c^2\pi(7a + 17b) = 440000.$

33. $r\left(\sqrt{\frac{3n}{m} - \frac{3}{4}} - \frac{1}{2}\right) = 2(\sqrt{205} - 1) = 26{,}6391;\ 4 : 1.$

34. $0{,}0582913.$ **35.** 16 und 4^{cm}.

36. $\frac{R^3 + 2R^2r + 2Rr^2 + r^3}{R^3 + r^3} = \frac{61}{21}.$ **37.** $\frac{1}{3}r^3\pi(3\sqrt{2} + 2) = 1000.$

38. $R = r(\frac{2}{3}\sqrt{3} + \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{6});\ \varrho = \frac{1}{2}r(\sqrt{6} - \sqrt{2});$
$h = 2r(1 + \sqrt{\frac{2}{3}});\ s = r(\sqrt{6} + 2);$
$M = 2r^2\pi(3 + \sqrt{2} + \frac{2}{3}\sqrt{3} + \sqrt{6});$
$V = \frac{2}{3} r^3\pi(7 + 3\sqrt{2} + 2\sqrt{3} + \frac{22}{9}\sqrt{6}).$

39. $Gh - \sqrt{G\pi} \,.\, h^2 \operatorname{cotg} \alpha + \frac{1}{3}\pi h^3 \operatorname{cotg} \alpha^2 = 3{,}5.$

40. $\frac{1}{4}\pi s \,.\, \sin\alpha \,.\, \left\{\frac{M^2}{s^2\pi^2} + \frac{1}{3} s^2 \cos\alpha^2\right\} = 1000.$

41. $R = \sqrt{\frac{V}{\pi d} \cdot \operatorname{cotg} \alpha - \frac{1}{12} d^2} + \frac{1}{2} d = 6;$
$r = \sqrt{\frac{V}{\pi d} \cdot \operatorname{cotg} \alpha - \frac{1}{12} d^2} - \frac{1}{2} d = 2.$

42. $\frac{M \,.\, \sin\alpha \,.\, (p^2 + pq + q^2)}{3(p + q)} \cdot \sqrt{\frac{M\cos\alpha}{\pi(p^2 - q^2)}};\ \alpha)\ 400;\ \beta)\ 357.$

43. $\frac{2}{3}\pi \cdot \frac{R^3 - r^3}{\operatorname{cotg}\alpha + \operatorname{cotg}\beta} = 800.$

44. $\operatorname{tang}\alpha = \frac{24 K \pi^2}{P^3 - p^3};\ \alpha = 40^0.$

45. $\left(\frac{F^2}{c} + \frac{1}{12} c^3 \operatorname{cotg}\alpha^2\right)\pi = 10000.$

46. $\sqrt[3]{\frac{3V}{7\pi}} \cdot \operatorname{cotg}\frac{1}{2}\alpha^2 = 1.$

47. $\frac{m^2(\operatorname{cotg}\frac{1}{2}a^2 + 3\operatorname{cotg}\frac{1}{2}\alpha + 3)}{n^2\operatorname{cotg}\frac{1}{2}a^2 + 3mn\operatorname{cotg}\frac{1}{2}\alpha + 3m^2} = \frac{1}{3}.$

§. 26.

1. α) 2000; β) 7650; γ) 2659750000 Cub.-Meilen.

2. $\sqrt[3]{3V : 4\pi}$; α) 5; β) 6); γ) 7.

3. $\frac{1}{6}\sqrt{O^3 : \pi}$; 0,03352; 0,2681; 523,6.

4. $2\sqrt[3]{4{,}5 \,.\, V^2\pi} = 555.$ **5.** $\sqrt[3]{r_1^3 + r_2^3} = 5.$

6. $\sqrt[3]{O_1^3 + 2O_1O_2\sqrt{O_1O_2} + O_2^3} = 36.$

7. $\frac{4}{3}\pi\sqrt{\left(a^2 + \frac{u^2}{4\pi^2}\right)^3} = 3640.$

8. 1,55514 Kgr. **9.** 0^{dc},92638. **10.** 2,0348 Grm.

11. $r = \frac{1}{2}d = \sqrt[3]{\frac{3a(1-s)}{4\pi(s_1 - 1)}} = 4^{cm},265.$

12. $r(1 - \sqrt[3]{\frac{69}{74}}) = 0^{dc},0461.$

13. $\frac{1}{2}d\left(1 - \sqrt[3]{1 - \frac{s}{s_1}}\right) = 1^{dc},67325;$
Gewicht: $\frac{1}{6}\pi d^3 s = 1000$ Kgr.

14. $6a : d^3 s\pi = 2620.$ **15.** 0^{dc},25.

16. $R - \sqrt[3]{R^3 - r^3 + (r-a)^3} = 3.$

17. $R = \sqrt{\frac{a}{4b\pi} - \frac{b^2}{12}} + \frac{b}{2} = 3;\ r = \sqrt{\frac{a}{4b\pi} - \frac{b^2}{12}} - \frac{b}{2} = 1.$

18. $\frac{\pi}{3d}(3\varrho^4 + d^4).$

19. $R : r = (\sqrt{6} \pm 2) : 2;\ O : o = (5 \pm 2\sqrt{6}) : 2;$
$V : v = (9\sqrt{6} \pm 22) : 4.$

20. $4r^3(2 - \frac{1}{3}\pi) = 100.$ **21.** 1 : 2 : 3.

22. $r\sqrt[3]{12}.$ **23.** 4 : 7; 2 : 9.

24. $\frac{1}{3}\sqrt{O:\pi} = 1{,}13.$ **25.** $3(2m - n) : 4n;\ 7 : 6.$

26. $h\sqrt[3]{6K\pi^2} = 1{,}79.$ **27.** $\frac{1}{3}r\sqrt{3}.$

28. $x^3 + 3bx^2 = 12a : \pi;\ x^3 + 15x^2 = 14000;\ x = 20.$

29. $x^3 - 6ax^2 + 4a^3 = 0.$ Setze $x = z \cdot a;$
$z^3 - 6z^2 = -4;\ z = y + 2;\ y^3 - 12y - 12 = 0;$
y nahezu gleich $-1{,}1158;\ x = 3{,}537.$

30. $r^3\pi(\frac{4}{3} - \frac{1}{2}\sqrt{2}) = 8.$ **31.** $\frac{4}{3}\pi(r^2 - \varrho^2)^{\frac{3}{2}}.$

32. $\frac{1}{3}O\sqrt{6} = 89.$ **33.** $\frac{1}{12}d^2 s\pi(3h - d) = 225$ Kgr.

34. $\frac{1}{2}r\sqrt{2}.$ **35.** $h + \frac{2a^3}{3d^2} = 2.$

36. $O_1 : O_{11} = (m^2 + n^2)^2 : mn^2(m + \sqrt{m^2+n^2}) = 625 : 384;$
$V_1 : V_{11} = (m^2 + n^2)^3 : 2m^2n^4 = 15625 : 4608.$

37. $r\sqrt[3]{4(n^2 - 1)} = 12.$ **38.** $4 : 9.$

39. $32 : 9.$ **40.** $5929 : 780.$

41. $\frac{4}{3}\pi \cdot \left[\frac{1}{h}\sqrt{\frac{3V}{\pi h}} \cdot \left(\sqrt{\frac{3V}{\pi h} + h^2} - \sqrt{\frac{3V}{\pi h}}\right)\right]^3 = 3{,}22643.$

42. $h = 24$ oder $8;\ r = \sqrt{12}$ oder $6.$

43. $9 : 4 : 3;\ 18 : 8 : 3\sqrt{2}.$ **44.** $1 : 2.$

45. $\frac{1}{3}\pi \cdot \left[(s^2 - h^2)h - \frac{4}{h^3}(s \cdot \sqrt{s^2 - h^2} - s^2 + h^2)^3\right] = 19877.$

46. $h^4 : h\left[h - \frac{2r}{h}(\sqrt{r^2 + h^2} - r)\right]^3 : 4r\,[\sqrt{r^2 + h^2} - r]^3 =$
$1 : (9 - 4\sqrt{5}) : 2(\sqrt{5} - 2) = 1 : 0{,}0557281 : 0{,}4721359.$

47. $\frac{2\pi}{3} \cdot \frac{r^2h^3}{4r^2 + 3h^2};\ 2 : 1.$

48. $(2n - \sqrt{n^2 + 4}) : \sqrt{n^2 + 4};\ 1 : 5;\ 11 : 13.$

49. $r\sqrt[3]{15} = 15.$

50. $\varrho = r\sqrt[3]{\frac{4n}{m}};\ h = \frac{m}{n}r\sqrt[3]{\frac{4n}{m}};\ 4 : 1.$

51. $n(2m + n) : 2(m + n)^2 = 33 : 98,$
bezw. $m(2n + m) : 2(m + n)^2 = 20 : 49.$

52. $\frac{20}{9}F\sqrt{\frac{F}{3\pi}} = 32{,}2454;$
$\frac{2}{3}F(5 - \sqrt{3}) = 2{,}17863\ F = 27{,}3781.$

53. $2\sqrt{5} : 5,\ 2\sqrt{3} : 3$.

54. Sind R, h, s bezüglich die Längen des Radius, der Höhe und der Seitenlinie des Kegels, so ist $\varrho = \frac{h^2 R}{s(R+h)}$ und das gesuchte Verhältniss gleich $4Rh\sqrt{R^2 + h^2} : (R + h)^3$.

55. $4 : 1$. 56. $h^3 - 2rh^2 + \frac{4}{n} r^3 = 0$;
$h^3 - 12h^2 + 243 = 0,\ h_1 = 9;\ h_2 = \frac{1}{2}(\sqrt{117} + 3) = 6{,}90835$.

57. $\frac{1}{2}\, a(\sqrt{3} + 1) = 571$. 58. $32(\sqrt{5} - 2) : 5$.

59. $\sqrt{\frac{h}{12} \cdot \frac{R^2 + Rr + r^2}{R + r} - \frac{(R + r)^2}{12}} = A$;
$x = \frac{R+r}{2} - A = 4;\ y = \frac{R+r}{2} + A = 6$.

60. $O = 4\pi^2;\quad V = \frac{4}{3}\,\pi^2\sqrt{\pi}$.

61. $R_1 = \frac{1}{2}\, r(\sqrt{2n + 1} + \sqrt{2n - 3}) = 4$;
$R_2 = \frac{1}{2}\, r\, (\sqrt{2n + 1} - \sqrt{2n - 3}) = 1$.

62. $4 : 7$.

63. $\left.\begin{matrix} R \\ r \end{matrix}\right\} = \frac{1}{8}\, d\left(\sqrt{\frac{11 + \sqrt{105}}{2}} \pm \sqrt{\frac{31 - 3\sqrt{105}}{2}}\right) = \begin{matrix} 2{,}71442 \\ 2{,}17463. \end{matrix}$

64. $\frac{11}{81}\, R^3\pi$. 65. $(r^2 + 3a^2) : 3(r^2 - a^2) = 1 : 2$.

66. $5 : 2$. 67. $r = \frac{3}{4}\, a\sqrt{2}$.

68. $\frac{a}{2}\sqrt{2 - \sqrt{2}} \cdot \sqrt[3]{\frac{3(\sqrt{2} + 1)}{\pi}} = 5{,}0554$.

69. $6^{cm}{,}4320$; $1^{cm}{,}9563$. 70. 2,5632 Kgr.

71. $\frac{1}{106} : \frac{2}{89} : \frac{\sqrt{3}}{76}$. 72. $1 : 1 : \sqrt{3}$.

73. $V = \frac{1}{8}\, a^3;\ \varrho = \frac{1}{12}\, a\sqrt{51};\ O = \frac{17}{12}\, a^2\pi$;
$V_1 = \frac{17}{432}\, a^3\sqrt{51}$.

74. $9\sqrt{2} : \pi;\ \frac{3}{1};\ \frac{31}{10};\ \frac{65}{21};\ \frac{96}{31};\ \frac{353}{114}$.

75. $\frac{27}{26}\, V;\ \frac{3\sqrt{3}}{13\pi}\, V$.

76. $\frac{3}{26}\, V\, (9 + \sqrt{3});\ \frac{3V}{26\pi}\, (9 + \sqrt{3})$.

77. Wie $\sqrt{3} : 1$. Die Grundflächen der Cylinder theilen jedesmal die Diagonalaxe des Würfels in 3 gleiche Theile und berühren die Diagonalen der Würfelflächen.

78. $7:4$. **79.** $\frac{1}{3}a\sqrt[3]{4\sqrt{6}}$. **80.** $3:1$.

81. $(6\sqrt{3}-10):1$. **82.** 2654,2.

83. $r\sqrt[3]{\frac{3\pi+1}{2\pi}}$. **84.** 8023.

85. $\frac{1}{2}\left[\sqrt[3]{\frac{3}{4\pi}(ac+b)\pm\sqrt{(ac+3b):\sqrt[3]{48\pi^2(ac+b)}}}\right]=5;\ 4.$

86. $\cos\alpha=1-F\sqrt[3]{2:9V^2\pi};\ \alpha=36^0;$

$b=\sqrt[3]{6V\pi^2\cdot\frac{\alpha}{360^0}}=3{,}76991.$

87. $\frac{1}{2}V.\sin\alpha^2.\cos\frac{1}{2}\alpha^2=800.$

88. $\sqrt[3]{\frac{1}{4}(R^3-r^3)\operatorname{tang}\alpha}=8{,}796.$

89. $\frac{1}{4}s^3\pi.\sin\alpha=1000;\ 3:2\sin\alpha^2=1:0{,}47426.$

90. $2r\pi:\sqrt[3]{\frac{1}{2}\sin\alpha^2.\cos\alpha}=100.$

91. $V=\frac{1}{6}a^3\pi\left(\operatorname{cotg}\frac{180^0}{n}\right)^3(\sqrt{m^2+1}-1)^3:m^3=4{,}0662\,a^3;$

$O=a^2\pi(m^2\cos\alpha^2+1)^2:m^2\sin2\alpha^2=67{,}635\,a^2.$

92. $\frac{1}{6}\pi c^3\sqrt{(1+\cos\alpha^2.\operatorname{tang}\beta^2)^3}=42.$

93. Die Seiten der Grundfläche sind gleich 40; 13; 37; der Inhalt derselben ist gleich 240, die ganze Oberfläche gleich 865,8; $V=2395{,}5$.

94. $\frac{1}{6}a^3(1+\operatorname{tang}\alpha)[\pi(1+\operatorname{tang}\alpha)^2-6]=500.$

§. 27.

1. $\frac{2}{3}\pi(\varrho^2+h^2)(\sqrt{\varrho^2+h^2}-h)=353{,}96.$

2. $\frac{1}{3}r^3\pi;\ 1:4.$ **3.** $\frac{m^2}{n^2}V=68{,}6.$

4. $\frac{P^2}{12\pi^2}(P\mp\sqrt{P^2-p^2});\ \frac{P^3}{30\pi^2}$ oder $\frac{2P^3}{15\pi^2}$.

5. $\frac{1}{3}\pi h^2(3r-h);\ \alpha)\ 300;\ \beta)\ 400.$

6. $\frac{V}{\pi h^2}+\frac{1}{3}h=5.$

7. $h^3-3rh^2+\frac{3V}{\pi}=0;\ h^3-6h^2+5=0;\ h=1.$

8. $\frac{1}{6}\pi h(3\varrho^2+h^2)=80.$

9. $\frac{1}{3}\pi h^2\left(\frac{3}{3}\sqrt{\frac{O}{\pi}}-h\right)=20.$

10. $\frac{a^3 - 3ah^2 + 2h^3}{a^3} = 0{,}896.$

11. $\frac{1}{24\pi^2} \cdot [2b^3 + (2b^2 + a^2)\sqrt{b^2 - a^2}] = 52{,}7$ Grm.

12. $\frac{9}{8} r^3\pi \,.\, 1{,}15$ gr. $= 29{,}9$ Kgr.

13. $\frac{1}{6}\pi \,.\left[\left(\frac{O}{\pi} - a^2\right)\sqrt{\frac{2O}{\pi} + a^2} \mp a^3\right] = 2200{,}6;\ 5100{,}11.$

14. $h = \frac{3}{2} r = 9{,}50765;\ V = \frac{9}{8} r^3\pi = 900.$

15. $\frac{1}{2} r(3 - \sqrt{5}).$ **16.** $\sqrt[3]{\frac{3Vn^3}{4\pi(n-1)^2(n+2)}} = \sqrt[3]{\frac{3V}{2\pi}} = 7.$

17. $\frac{4}{3} r^3\pi \cdot \frac{m^2(m+3n)}{(m+n)^3} = 435{,}63;\ \frac{4}{3} r^3\pi \,.\, \frac{n^2(3m+n)}{(m+n)^3} = 3753{,}18.$

18. $x^3 - 3r^2x + 2r^3 \cdot \frac{n-m}{n+m} = 0;$
$x^3 - 3r^2x + \frac{11}{8} r^3 = 0;\ x_1 = \frac{1}{2} r.$

19. $d = \frac{1}{2} r;\ V = \frac{5}{24} r^3\pi.$

20. $r = \sqrt{\frac{O}{6\pi}} = 3;\ V = \frac{9}{4} r^3\pi = 190{,}852.$

21. $r = \frac{1}{2}\sqrt{a^2 + b^2 + c^2} = 6{,}5;$
$\frac{1}{3}\pi(r - \frac{1}{2}a)^2 \,.\, (2r + \frac{1}{2}a) = 379{,}61;$
$\frac{1}{3}\pi(r + \frac{1}{2}a)^2 \,.\, (2r - \frac{1}{2}a) = 770{,}74.$

22. $7 : 20.$ **23.** $(43 - 24\sqrt{3}) : 11.$ **24.** $(43 - 24\sqrt{3}) : 11.$

25. $\frac{4}{9} a^3\sqrt{2} - \frac{2}{81} a^3\pi(14\sqrt{6} - 27).$

26. $\frac{1}{2} r^3\pi\sqrt{2};\ \frac{1}{3} r^3\pi\sqrt{2}$ und zweimal $\frac{1}{12} r^3\pi(8 - 5\sqrt{2}).$

27. $\frac{1}{6} r(9 - \sqrt{33}).$

28. $\frac{4}{3} r^3\pi\left(1 - \frac{n-m}{n}\sqrt{\frac{n-m}{n}}\right) = \frac{9}{2}\pi(8 - 3\sqrt{3}) = 39{,}639.$

29. $\frac{1}{3} h^2\pi\left(3\sqrt{\frac{O}{6\pi}} \cdot \sqrt[3]{\frac{3}{2}} - h\right) = 20.$ **30.** $\sqrt[3]{2{,}8} : 2.$

31. $\frac{1}{2} r(\sqrt{3} - 1).$ **32.** $8 : (8 - 3\sqrt{3}).$ **33.** $1^{dc},04.$

34. $V = \frac{1}{12}\pi \,.\, [(a^2 + ab + b^2)c + (\frac{3}{2}b^2 + 2h^2)h] = 9550;$
$O = \pi \cdot \left(\frac{b^2}{4} + h^2 + \frac{a+b}{2} \cdot \sqrt{c^2 + \frac{(a-b)^2}{4}}\right) = 1820{,}33.$

35. $(2a^3 - 3a^2r + r^3) : (2a^3 + 3a^2r - r^3) = 13 : 112.$

36. $r = \frac{V}{\pi h^2} + \frac{h}{3} = 3{,}19433$; $x = \frac{4r^3}{2rh - h^2} = 17{,}0362$;
$s = \sqrt{x^2 + 2rh - h^2} = 17{,}2592$;
$M = \sqrt{2rh - h^2} \,.\, s\pi = 150$; (149,997).

37. $\frac{4}{3} r^3 \pi \cdot \frac{(p^2 - q^2)^2 \,.\, (p^2 + 2q^2)}{p^6} = 9923.$

38. a) $\frac{r^2\pi}{(p-q)^2} \sqrt{q(8p^3 - 32p^2q + 42pq^2 - 18q^3)}$
$= \frac{25}{9}\pi\sqrt{30} = 47{,}798$; b) $2r^2\pi \,.\, \frac{2p - 3q}{p - q} = \frac{50}{3}\pi = 52{,}36.$

39. $r \cdot \frac{\sqrt{17} + 1}{4} = 4.$ **40.** $\frac{4R^3r^2\pi}{3(4R^2 + r^2)^3}(32R^4 + 12R^2r^2 + r^4).$

41. $9 : 25.$ **42.** $\frac{5}{12} r^3\pi$; $5 : 16.$ **43.** $\frac{1}{3} r.$

44. $r - \sqrt[3]{r^3 - \frac{1}{4}h^2(3r - h)} = \sqrt{3}.$

45. $(2m + n - 3\sqrt[3]{m^2 n}) : 4n = 1 : 4.$

46. $x^3 - 3r^2x + r^3 = 0.$

47. $r = (\frac{1}{4}d^2 + h^2) : 2h = 9{,}51175$; $x = r - h = 4{,}75585$;
$R = \sqrt{(a + \frac{1}{2}d)^2 + x^2} = 11{,}9689$; $H = R - x = 7{,}2130$;
$V = \frac{1}{3}\pi[H^2(3R - H) - h^2(3r - h)] = 1000$ (nahezu).

48. $r\sqrt{\frac{n - m}{n}} = \frac{r}{5}\sqrt{21}.$

49. $2r^3\pi(66\sqrt{5} - 125) : 75 = V$; $\frac{4}{3}r^3\sqrt{3} - V.$

50. $\frac{1}{3}\pi h \,.\, [3r^2 - 3p^2 - h^2 \mp 3hp] = 0{,}42564$; 0,3.

51. $\frac{1}{3}\pi(\sqrt{R^2 - \varrho^2} \mp \sqrt{R^2 - r^2}) \,.\, [R^2 + r^2 + \varrho^2 \mp \sqrt{(R^2 - \varrho^2)(R^2 - r^2)}]$;
17507 und 44277.

52. $\frac{1}{3}\pi h[3r^2 - h^2 \mp \frac{3}{2}(r^2 - \varrho^2 - h^2)] = 285463$ und 142250.

53. $(3r^2 - h^2) : 2h(3r - 2h) = 13 : 12.$

54. $\sqrt{r^2 - \frac{1}{12}h^2 - \frac{4r^3}{3nh}} \mp \frac{1}{2}h = \frac{1}{2}r.$

55. $h\pi(\varrho^2 - \frac{1}{12}h^2)$; $(\varrho^2 + \frac{1}{4}h^2) : h.$

56. $236 : 7.$ **57.** $a^3 = \frac{66}{125}r^3\pi = 677{,}1$; $a = 8{,}7812.$

58. $\frac{3}{2}a^2h\sqrt{3} + \frac{1}{3}mh(\frac{9}{4}a^2 + \frac{3}{2}a\sqrt{f} + f) +$
$\frac{h\pi}{24} \cdot [3a^2(5 + 6n) - 2h^2(1 + 3n) + a\sqrt{9a^2 - 3h^2}].$

59. Die Zweiecke $= \frac{1}{12}a^3[1 + \pi(\sqrt{3} - 2)]$,
die Abschnitte $= \frac{1}{2}a^3[\pi(4 - \sqrt{3}) - 4].$

60. $\frac{4}{3} r^3 \pi \,.\, \sin \frac{1}{4} \alpha^2 = 3320.$ **61.** $\sqrt[3]{3V : 4\pi \,.\, \sin \frac{1}{4} \alpha^2} = 10.$

62. $V . \sin \frac{1}{4} \alpha^2 = 19.$ **63.** $4 h^3 \pi \,.\, \sin \frac{1}{2} \alpha^2 : 3 \cos \alpha^3 = 10.$

64. $\sqrt{\frac{b}{4\pi \sin \frac{1}{4} \alpha^2} - \frac{1}{12} a^2} - \frac{1}{2} a = 30.$ **65.** $103^0\, 39'\, 15'',6.$

66. $\cos \varphi = \cos \frac{1}{2} \alpha^2;\ \varphi = 60^0.$

67. 34962300 Kubik-Meilen.

68. $\frac{1}{3} r^3 \pi \,.\, (1 - \cos \frac{1}{2} \varphi)^2 \,.\, (2 + \cos \frac{1}{2} \varphi) = 35.$

69. $\cos \alpha^3 - 3 \cos \alpha = \frac{4V}{K} - 2 = - \frac{11}{8};\ \cos \alpha = \frac{1}{2},\ \alpha = 60^0.$

70. $137^0\, 3'\, 30'';\ \sqrt{3} : 1.$

71. $r^3 \pi\, (1 - \sin \frac{1}{2} \alpha)^2 : 3 \sin \frac{1}{2} \alpha = 48,9.$

72. $\alpha = \frac{m}{m+n} \cdot 180^0;\ V = \frac{4}{3} r^3 \pi (1 + \cos \alpha - \cos \alpha^2)$
$= \frac{5}{3} r^3 \pi = 4000.$

73. $\frac{2}{3} r^3 \pi \,.\, \sin \frac{1}{2} \alpha \,.\, (3 - \sin \frac{1}{2} \alpha^2) = 510.$

74. $\frac{1}{3} r^3 \pi \,.\, [3 \cos \frac{1}{2} \beta - 3 \cos \frac{1}{2} \alpha - \cos \frac{1}{2} \beta^3 + \cos \frac{1}{2} \alpha^3] = 85.$

75. $V_1 = V + \frac{1}{6} \pi h^3 \,.\, \operatorname{cosec} \alpha^2 = 351,308;$
$V_2 = V - \frac{1}{6} \pi h^3 (5 - \operatorname{cotg} \alpha^2)$
$+ 2\pi h^2 \operatorname{cotg} \alpha \sqrt{\frac{V}{\pi h} - \frac{1}{12} h^2 \operatorname{cotg} \alpha^2} = 512,038.$

76. $\frac{1}{3} h^3 \pi \sin \alpha^4 \operatorname{tang} \alpha^2.$

§. 28.

1. $\frac{2}{3} r^3 \alpha \cdot \frac{\pi}{180^0} = 43,8.$ **2.** $\sqrt[3]{\frac{3V}{2\alpha} \cdot \frac{180^0}{\pi}} = 1.$

3. $\frac{3V}{2r^3} \cdot \frac{180^0}{\pi} = 60^0.$ **4.** $\frac{r^3 \pi}{3} \cdot \frac{\alpha + \beta + \gamma - 180^0}{180^0} = 70,5.$

5. $\frac{1}{3} r^3 \pi \,.\, [\alpha + \beta + \gamma + \delta + \ldots - (2n - 4) 90^0] : 180^0.$

6. $a^3 (\frac{1}{24} \sqrt{2} + \frac{13}{432} \pi \sqrt{6}) = 0,290500 a^3 = 2324.$

7. $\frac{1}{3} Fr.$ **8.** $\frac{1}{54} r^3 \pi (65 \sqrt{3} - 108) = 0,084876 r^3 \pi.$

9. $\sin \frac{1}{2} \alpha = m : n = 0,86;\ \alpha = 118^0\, 38';\ 1 : 6.$

10. $\sin \frac{1}{2} \alpha = \sqrt[3]{\frac{m}{n}} = \frac{1}{2};\ \alpha = 60^0;\ \sqrt[3]{\frac{m^2}{n^2}} : 1 = 1 : 4.$

11. $\frac{2}{3} r^3 \sin \frac{1}{2} \alpha \cdot \beta \cdot \frac{\pi}{180^0} = 4990.$

12. 1213,05. **13.** 932680.

14. $\frac{1}{3} r^3 \pi . [3(\cos\alpha + \cos\beta + \cos\gamma) -$
$(\cos\alpha^3 + \cos\beta^3 + \cos\gamma^3) - 2] = 18{,}4.$

15. $\frac{1}{3} a\pi . [3r^2(\sin\alpha + \sin\beta) - a^2(\sin\alpha^3 + \sin\beta^3)].$

§. 29.

2. $D_1 = -\frac{1}{6}h(a-b)^2$; $D_2 = \frac{1}{12}h(a-b)^2$; 4,15 und 2,07M.

3. $\frac{1}{2} h \cdot \left(\frac{a+a_1}{2} \cdot \frac{b+b_1}{2} + \frac{1}{3} \cdot \frac{a-a_1}{2} \cdot \frac{b-b_1}{2}\right)$;
α) 1029; β) 0,022.

4. $h \cdot \left(\frac{a+a_1}{2} \cdot \frac{b+b_1}{2} + \frac{1}{3}\frac{a-a_1}{2} \cdot \frac{b-b_1}{2}\right)$; α) $560\frac{2}{3}$; β) 54963.

5. $h \cdot \left[\left(\frac{a+a_1}{2}\right)^2 + \frac{1}{3}\left(\frac{a-a_1}{2}\right)^2\right]$; α) 148; β) 5244.

6. $h \cdot \left(\frac{a+a_1}{2} \cdot \frac{b+b_1}{2} + \frac{1}{3} \cdot \frac{a-a_1}{2} \cdot \frac{b-b_1}{2}\right)$; α) 1700; β) 68,04.

7. $O = 175{,}157$; $V = 149\frac{1}{3}$. 9. $\frac{29}{24} a^2 h \sqrt{3}$.

10. $\frac{a+b+c+d}{4} \cdot F$. 12. $\frac{1}{3} hG \cdot \frac{m^2 + mn + n^2}{m^2}$.

13. $\frac{3F(\sqrt{m} + \sqrt{n})^2}{4(m + \sqrt{mn} + n)} = 18\frac{3}{4}$ □cm.

15. $\frac{1}{6} b(2a + c)\sqrt{d^2 - \frac{1}{4}b^2} = 2{,}53$. 16. $\frac{3}{2} a^2 h \sqrt{3}$.

17. $\frac{2}{3} a^2 \sqrt{b^2 - \frac{1}{4}a^2}$. 18. $\frac{2}{3} abc$; $\frac{2}{3} a^3$. 19. $\frac{1}{6} a^3 \sqrt{2}$.

20. $\frac{1}{6} bh(a + c)$. 21. $\frac{1}{3} V$. 22. $\frac{1}{3} a^3 \sqrt{2}$.

23. $\frac{1}{2} a^2 h \sqrt{3} = 790$. 24. $\frac{1}{3} a^2 h(2 + \sqrt{2})$.

25. $\frac{1}{6} a^3(5 + 2\sqrt{5}) = \frac{1}{6} a^3 \operatorname{cotg} 18^{0\,2}$.

26. $\frac{1}{6} bh[4a + (3a^2 + b^2)(a^2 + b^2)^{-\frac{1}{2}}]$. 27. $\frac{5}{6} a^2 h$.

28. a) $\frac{1}{12} a^2 h(5 + 2\sqrt{2})$; b) $\frac{1}{3} h(a^2 + b^2 + ab\sqrt{2}) = 500$.

29. $\frac{5}{3} abh$. 30. $\frac{1}{3} h[a^2 + ac + c^2 + b^2 + (2a + c)b\sqrt{2}] = 836$.

31. $\frac{1}{4} ah(b + c + d) = 1080$. 32. $\frac{17}{8} a^2 h \sqrt{3}$.

33. $\frac{1}{3}\sqrt{3}(a^2 + ab + \frac{3}{2} b^2)\sqrt{c^2 - (a-b)^2}$.

34. $\pi h \left[\left(\frac{r + r_1}{2}\right)^2 + \frac{1}{3}\left(\frac{r - r_1}{2}\right)^2\right] = 4047$.

35. $2r\sqrt{\pi}$; $2r\sqrt{1 + \frac{1}{2}\pi}$.

36. $\frac{1}{3} h^2 \pi(3r - h)$; $\frac{1}{6} h\pi(3\varrho^2 + h^2)$.

37. $\frac{1}{2} h\pi(\varrho_1{}^2 + \varrho_2{}^2) + \frac{1}{6} h^3\pi$. 38. $\frac{1}{3} h\pi(r^2 + 2R^2)$.

39. $\frac{2}{3}\, r^2 s$. **40.** Zweimal $\frac{2}{3}\, r^2 h$ und $r^2 h(\pi - \frac{4}{3})$.

41. $\frac{1}{2}\, h \,.\, [(3ab + 3a'b' + 2ab' + 2a'b) \sin\alpha + (3cd + 3c'd' + 2cd' + 2c'd) \sin\beta] = 382289$. Nein; es können zwei Seiten des einen Vierecks aus den beiden anderen und den durch das zweite Viereck bestimmten Winkeln berechnet werden.

42. $\frac{1}{12}\, b^2 \,.\, \mathrm{tang}\,\beta \,.\, (3a + b \,\mathrm{cotg}\, \frac{1}{2}\,\alpha) = 5000$.

§. 30.

1. $O = a^2 \sqrt{3}\,\pi = 54{,}6525$; $V = \frac{1}{4}\, a^3 \pi = 25$.

2. $O = a\pi(a\sqrt{3} + 6b) = 74{,}7633$;
$V = \frac{1}{4}\, a^2 \pi (a + 2b\sqrt{3}) = 20$.

4. $O = \pi[a(b_1 + c_1) + b(c_1 + a_1) + c(a_1 + b_1)]$;
$V = \frac{1}{3}\,\pi(a_1 + b_1 + c_1) \cdot [p(c_1 - a_1) + q\,(c_1 - b_1)]$;
a) $a = \sqrt{(b_1 - c_1)^2 + p^2}$; $b = \sqrt{(c_1 - a_1)^2 + q^2}$;
$c = \sqrt{(b_1 - a_1)^2 + (p + q)^2}$; b) $p = \sqrt{a^2 - (b_1 - c_1)^2}$;
$q = \sqrt{b^2 - (c_1 - a_1)^2}$; $c = \sqrt{(b_1 - a_1)^2 + (p + q)^2}$;
c) $F = \frac{1}{2}\, p(c_1 - a_1) - \frac{1}{2}\, q(b_1 - c_1)$;
$V = \frac{2}{3}\,(a_1 + b_1 + c_1)\pi F$. Bei zu einer Seite paralleler Axe ist $O = \pi\,[(a + b)(a_1 + c_1) + 2\,a_1 c]$;
$V = \frac{1}{3}\,\pi(2a_1 + c_1)(c_1 - a_1)(p + q)$, und für $a_1 = 0$ ist $O = \pi(a + b)\,.\,c_1$; $V = \frac{1}{3}\,\pi c_1{}^2(p + q)$.

5. $\frac{a+b}{c} : \frac{a+c}{b} : \frac{b+c}{a}$; $\frac{1}{c} : \frac{1}{b} : \frac{1}{a}$. **6.** $2a\pi \,.\, F = 2000$.

7. $\frac{4ab\pi(a+b)}{\sqrt{a^2+b^2}} = 7461{,}7$; $\frac{2a^2b^2\pi}{\sqrt{a^2+b^2}} = 38219$.

8. $a^3\pi = 800$. **9.** $\frac{7}{12}\, a^3\pi\sqrt{3} = 77$.

10. $\frac{a^3\pi}{2} \cdot \frac{2m^2 + 3mn + 2n^2}{(m+n)\sqrt{m^2 + mn + n^2}}$.

11. $\frac{1}{6}\, a^3\pi(4 + 3\sqrt{2})\sqrt{2 + \sqrt{2}}$; $\frac{1}{12}\, a^3\pi(15 + 11\sqrt{2})$.

12. $O = \frac{7}{4}\, a^2\pi$; $V = \frac{1}{8}\, a^3\pi\sqrt{3}$; Ja!

13. $O = \frac{5}{2}\, a^2\pi\sqrt{2}$; $V = \frac{1}{2}\, a^3\pi\sqrt{2}$.

14. $7a^2\pi$; $a^2\pi$; $\frac{3}{2}\, a^3\pi\sqrt{3}$. **16.** Vergl. 4c.

17. $\sqrt[3]{m} : (\sqrt[3]{m} - \sqrt[3]{n}) = 1 : 1$.

18. $O = 6a^2\pi\sqrt{3}$; $V = \frac{9}{2}\, a^3\pi$.

19. $O = \pi\sqrt{c^2 - \frac{1}{4}(a-b)^2} \,.\, (a^2 + b^2 + ac + bc) : c;$
$V = \frac{1}{3}\pi(a^2 + ab + b^2)(c^2 - \frac{1}{4}[a-b]^2) : c.$

20. $(2n - m) : (2m - n) = 5 : 2.$

21. $\frac{1}{24}a^3\pi(7 - 4\sqrt{2}) = 10.$

22. $\frac{1}{3}\pi \,.\, a^2b^2 \sin\gamma^2 : \sqrt{a^2 + b^2 - 2ab\cos\gamma} = 40{,}384.$

23. $O = \dfrac{a^2\pi \,.\, \sin\beta \,.\, \sin\gamma \,.\, \cos\frac{1}{2}(\beta - \gamma)}{\sin\alpha \,.\, \cos\frac{1}{2}(\beta + \gamma)} = 9{,}0456;$
$V = \frac{1}{3}\pi \,.\, \dfrac{a^3 \sin\beta^2 \sin\gamma^2}{\sin\alpha^2} = 1{,}527.$

24. $O = \pi c \sin\alpha(c + 2b + \sqrt{(a-b)^2 - 2(a-b)c\cos\alpha + c^2}) = 6{,}648;$
$V = \frac{1}{3}\pi c^2 \,.\, \sin\alpha^2 \,.\, (a + 2b) = 0{,}72.$

25. $\varrho = \dfrac{a\sin\beta\sin\gamma}{\sin\alpha} - e;\ x = \dfrac{\varrho}{\sin\beta},\ y = \dfrac{ae}{\varrho + e},\ z = \dfrac{\varrho}{\sin\gamma};$
$O = \varrho\pi(x + 2y + z) = 3{,}5665;$
$V = \frac{1}{3}\varrho^2\pi(a + 2y) = 0{,}7015.$

26. $O = 2a^2\pi\sqrt{3} \,.\, \sin(30^0 + \alpha) = 275{,}85;$
$V = \frac{1}{2}a^3\pi \,.\, \sin(30^0 + \alpha) = 250{,}02.$

27. $\frac{4\pi}{3}(1 + \cos\alpha^2)\sqrt{\dfrac{F^3}{\sin 2\alpha}} = 6328{,}7.$

28. $O = \dfrac{ab\pi\sin\gamma}{2m}(2a + b + 3c) = 42922;$
$V = \dfrac{a^2b^2\pi\sin\gamma^2}{2m} = 339520.$

29. $r = a \,.\, \sin(\alpha + \gamma - \delta) = 2{,}00167;$
$\varrho = b \,.\, \sin(\alpha - \delta) = 1{,}00166;$
$O = \pi[(a + c)r + (b + c)\varrho] = 984{,}96;$
$V = \frac{1}{3}\pi(r + \varrho)ab \,.\, \sin\gamma = 355{,}77.$

30. $2F \,.\, d\pi.$ **31.** $2U \,.\, d\pi.$ **32.** $2ar^2\pi^2;\ 4ar\pi^2.$

33. $O = 2r^2\pi \,.\, (2\sin\frac{1}{2}\alpha + \cos\frac{1}{2}\alpha) = 147{,}92;$
$V = \frac{4}{3}r^3\pi \,.\, \sin\frac{1}{2}\alpha = 80.$

34. $O = 4r^2\pi \,.\, \sin\frac{1}{2}\alpha(1 + \cos\frac{1}{2}\alpha);$
$V = \frac{4}{3}r^3\pi \,.\, \sin\frac{1}{2}\alpha^3.$ Für $\alpha = 180^0$ ist $O = 4r^2\pi$, $V = \frac{4}{3}r^3\pi$; für $\alpha = 90^0$ ist $O = 2r^2\pi(\sqrt{2} + 1)$, $V = \frac{1}{3}r^3\pi\sqrt{2}$; für $\alpha = 60^0$ ist $O = r^2\pi(2 + \sqrt{3})$, $V = \frac{1}{6}r^3\pi.$

35. $O = s\pi\cos\alpha \,.\, (2r + \sqrt{4r^2 - s^2});$
$V = \dfrac{s^3\pi}{6}\cos\alpha.$

36. $\frac{1}{8} a^3 \pi^2$.

37. $\frac{r^3 \pi}{3} \sin \alpha^2$; $\frac{4 r^3 \pi}{3} \sin \frac{1}{2} \alpha^4$; $\frac{2 r^3 \pi}{3} \sin \frac{1}{2} \alpha^2 \operatorname{tang} \frac{1}{2} \alpha^2$; $\alpha = 90^0$; $1 : 1 : 4$.

§. 31.

1. Der grösste hat den durch jenen Punkt gehenden Durchmesser, der kleinste die auf letzterem senkrechte Sehne zur Grundlinie.

2. Man ziehe diejenigen Sehnen von der gegebenen Länge, welche zu dem Neigungsschenkel senkrecht sind, und lege durch sie die Schnitte.

3. Er ist ein regelmässiges Sechseck und halbirt den Abstand der beiden parallelen Oktaëderflächen.

4. Er ist ein regelmässiges Sechseck und halbirt den Abstand der beiden parallelen Schnittflächen.

5. Er ist ein Quadrat, und seine Eckpunkte halbiren vier Kanten des Tetraëders.

6. Seine Grundfläche ist ein Quadrat.

7. Der Radius der Grundfläche des Kegels ist gleich r, die Seitenlinie gleich $3r$; Grundfläche und Mantel theilen sich also in die gegebene Oberfläche im Verhältniss $1 : 3$.

8. $\frac{2}{27} a^3 \pi \sqrt{3}$.

9. Die Höhe des Cylinders ist gleich dem Durchmesser seiner Grundfläche, der Axenschnitt also ein Quadrat. $r = \sqrt{O : 6\pi}$.

10. Wie vorher. $r = \sqrt[3]{V : 2\pi}$.

11. Der Radius des Grundkreises ist gleich $\frac{1}{3} r \sqrt{6}$, die Höhe gleich $\frac{2}{3} r \sqrt{3}$. Durchmesser des Grundkreises und Höhe verhalten sich also wie Diagonale und Seite eines Quadrats. Das dem Cylinder einbeschriebene gerade quadratische Prisma ist ein Würfel.

12. r^3.

13. $r = \frac{1}{6} d \sqrt{6}$, $h = \frac{1}{3} d \sqrt{3}$, $V = \frac{1}{18} d^3 \pi \sqrt{3}$.

14. Die Höhe des Kegels ist gleich $\frac{4}{3} r$, sein Schwerpunkt fällt also mit demjenigen der Kugel zusammen.

15. $\frac{64}{81} r^3$; $(a = h.)$ **16.** $\varrho = \frac{1}{3} r$.

17. Er hat den grössten Mantel, aber nicht die grösste Gesammt-Oberfläche.

18. Die Höhe des Kegels, welcher den kleinsten Mantel hat, ist gleich dem Durchmesser einer Kugel von gleichem Rauminhalt. Es verhalten sich die Quadrate des Radius, der Höhe und der Seitenlinie des Kegels wie $1 : 2 : 3$.

19. $\sqrt[3]{2V} = 2$; $\sqrt[3]{\frac{1}{4} V} = 1$.

20. $x = y = \sqrt{V : h} = 3^{cm}$; $40 \left(\frac{V}{h} + 2 \sqrt{Vh}\right)$ Pf. $= 18$ M.

21. Das kleinste Volumen hat ein Kegel, dessen Höhe gleich dem doppelten Durchmesser der Kugel ist. Der Durchmesser seiner Grundfläche verhält sich zur Höhe, wie die Seite eines Quadrats zu seiner Diagonale. Das Volumen des Kegels ist doppelt so gross als das der Kugel. Derselbe Kegel hat die kleinste Gesammt-Oberfläche; dagegen ist für denjenigen, welcher den kleinsten Mantel hat, die Höhe gleich $r \,.\, (2 + \sqrt{2})$.

22. $h = \frac{2}{3} r = \frac{1}{9} a \sqrt{6}$.

23. Der Radius der Grundfläche ist gleich $\frac{3}{2} r$, die Höhe gleich $2h$, der Rauminhalt das $4\frac{1}{2}$fache des Volumens des gegebenen Kegels.

24. Der gesuchte Punkt schneidet von demjenigen Punkte aus, welcher die Spitze des entstehenden Kegels wird, $\frac{2}{3}$ der Hypotenuse ab.

25. a) Wie vorher. b) Ist die als Drehungsaxe dienende Kathete gleich a, die andere gleich b, so ist der Radius der Grundfläche des Cylinders gleich $\frac{ab}{2(a - b)}$, und es muss $b < \frac{1}{2} a$ sein.

26. Die Grundfläche ist gleich $\frac{4}{9}$ der Fläche des in die Kegelbasis einbeschriebenen regelmässigen Sechsecks, die Tiefe gleich $\frac{1}{3}$ der Kegelhöhe.

27. Die Centrallinie wird durch den Punkt im Verhältniss $R \sqrt{R} : r \sqrt{r}$ getheilt.

28. $\alpha = 360^0 \,.\, \sqrt{\frac{2}{3}} = 293^0\, 56'\, 24''$.

29. Die Höhe der Calotte ist gleich der Hälfte der Höhe der Pyramide.

§. 32.

11. Wie 1 : 2 und 1 : 1. Die Umfänge sind beide Male gleich $3a\sqrt{2}$, die Flächeninhalte bezüglich gleich $\frac{1}{4}a^2\sqrt{3}$ und $\frac{3}{4}a^2\sqrt{3}$.

12. Bezeichnet a die Länge einer jeden der längeren, b die der kürzeren Seite, c die von der anderen halbirte, d die halbirende Diagonale, sodass also $d = \sqrt{a^2 - \frac{1}{4}c^2} + \sqrt{b^2 - \frac{1}{4}c^2}$ ist, so sind die durch die Oktaëder-Ecken gehenden Axen gleich $c\sqrt{2} + 2\sqrt{a^2 - \frac{1}{2}c^2}$, die durch die Würfel-Ecken gehenden gleich $2 . (c\sqrt{\frac{2}{3}} + \sqrt{b^2 - \frac{1}{3}c^2})$, die durch die übrigen Ecken gehenden gleich $2c$. Für die Kantenwinkel zwischen den Seiten a ist $\sin\frac{1}{2}\alpha = c : 2a$, für diejenigen zwischen den Seiten b ist $\sin\frac{1}{2}\gamma = c : 2b$, für die übrigen $\beta = 90^0 - \frac{1}{2}(\alpha + \gamma)$. Für die Flächenwinkel an b ist $\sin\frac{1}{2}\varphi = f : 2b\sin\gamma$, für die Flächenwinkel an a ist $\sin\frac{1}{2}\psi = c\sqrt{2} : 2a\sin\alpha$. Die Oberfläche ist gleich $12cd$, das Volumen gleich $\frac{8c^2d}{\sqrt{4a^2-c^2}}(\frac{1}{2}c\sqrt{2} + \sqrt{a^2 - \frac{1}{2}c^2})$.

13. $(a + b)(c + d - 2h) . h$.

14. $O = ab\pi = 180\pi$; $V = \frac{1}{12}\pi(b - 2c)^2(3a - b - 4c) = \frac{15523}{48}\pi$.

15. $4ab^2 - b^3\sqrt{2}$. **16.** $9ab^2\sqrt{3}$.

17. a) Eine gerade quadratische Doppelpyramide (ein quadratisches Oktaëder), deren Randkanten gleich a, deren Seitenkanten gleich $\frac{1}{2}a\sqrt{3}$ sind, und deren Spitzen die Entfernung a von einander haben. $O = 2a^2\sqrt{2}$; $V = \frac{1}{3}a^3$.

b) Ein von zwölf Rhomben begrenzter Körper (ein regelmässiges Rhombendodekaëder); seine Kanten sind gleich $\frac{1}{4}a\sqrt{3}$, die längeren Diagonalen der Rhomben gleich $\frac{1}{2}a\sqrt{2}$, die kürzeren gleich $\frac{1}{2}a$; $O = \frac{3}{2}a^2\sqrt{2}$; $V = \frac{1}{4}a^3$.

18. Eine doppelte abgestumpfte Pyramide, deren Endflächen Quadrate mit bezüglich den Seiten a und $2a$, und deren Seitenflächen Trapeze mit den parallelen Seiten a und $2a$ und der Höhe a sind. Für den gemeinschaftlichen Körper ist $O = 14a^2$, $V = \frac{1}{3}a^3\sqrt{3}$, für das Kreuz $O = 2a(3a\sqrt{3} - 7a^2 + 6b)$, $V = a^2\sqrt{3}(3b - \frac{1}{3}a)$.

19. Ein gerades Prisma, dessen Grundfläche ein Quadrat mit der Seite $a\sqrt{3}$, und dessen Höhe gleich a ist, und zwei auf die Grundflächen aufgesetzte gerade Pyramiden, deren Höhen einzeln gleich $\frac{1}{2}a$ sind. Für den gemeinschaftlichen Körper ist $O = 8a^2\sqrt{3}$, $V = 4a^3$, für das Kreuz $O = 2a(6b - a\sqrt{3})$, $V = a^2(3b\sqrt{3} - 4a)$.

20. Ein gerades, quadratisches Prisma, auf dessen Grundflächen congruente gerade Pyramiden aufstehen. Die Höhe des Prismas ist gleich $\frac{1}{2}e$, seine Grundkante gleich $\frac{1}{2}d$, die Höhe einer Pyramide gleich $\frac{1}{4}e$, jede Seitenkante derselben gleich $\frac{1}{4}\sqrt{e^2 + d^2}$; $O = d \cdot e + \frac{1}{2}d \cdot \sqrt{e^2 + d^2}$; $V = \frac{1}{6}d^2e$. Für $d = e\sqrt{3}$ übereinstimmend mit 19.

21. a) Der Körper besteht aus zwei symmetrischen dreiseitigen Pyramiden, welche eine Seitenfläche gemeinsam haben; die Grundflächen sind gleichseitige Dreiecke mit der Seite a, die nicht gemeinschaftliche Seitenkante steht jedesmal senkrecht zur Grundfläche und ist gleich $\frac{1}{2}a\sqrt{3}$; $O = \frac{3}{2}a^2\sqrt{3}$; $V = \frac{1}{4}a^3$.

b) Eine gerade vierseitige Pyramide, deren Grundfläche ein Quadrat mit der Kante a, und deren Höhe gleich der Höhe der Grundflächen der Prismen ist. $O = 3a^2$; $V = \frac{1}{6}a^3\sqrt{3}$.

22. $O = 16r^2$; $V = \frac{16}{3}r^3$.

23. $\frac{1}{2}Op \cdot \left(\sqrt[3]{\frac{m}{n}} - 1\right) : \left(\sqrt[3]{\frac{m}{n}} + 1\right) = \frac{1}{6}Op$.

24. $\frac{4\pi}{3} \cdot \frac{R^2r^2}{R+r}$.

25. $\frac{\pi\varrho^2(a-\varrho)^2}{3a}$; $\frac{\pi}{3}\left[\frac{r^3}{\varrho}\sqrt{a^2 - \varrho^2} - \frac{\varrho^2}{a}(a+\varrho)^2\right]$;

$\varrho\sqrt{\frac{r^3\sqrt{32\varrho^6 + r^6} - 8\varrho^6 - r^6}{2(4\varrho^6 - r^6)}}$.

26. $(26 - 15\sqrt{3}) : 1 = 0{,}0194 : 1$ oder nahezu wie $97 : 5000$.

27. a) Ebenso, b) $1 : 8$.

28. $O = 12a^2(2 - \sqrt{2})$; $V = 2a^3(2 - \sqrt{2})$.

29. $O = \frac{3}{2}a^2(3 + 2\sqrt{6})$; $V = \frac{3}{2}a^3$.

30. $O = 3a^2(2 + 3\sqrt{3} - 2\sqrt{6})$; $V = \frac{5}{2}a^3(\sqrt{2} - 1)$.

31. $\frac{1}{12}a^3 \cdot (4\sqrt{2} - 1)$.